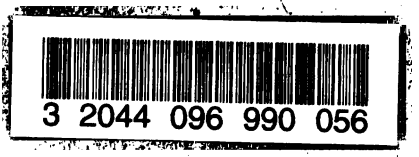
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JUNIOR
HIGH SCHOOL MATHEMATICS

SECOND BOOK

BY

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business arithmetic were omitted from the work of the seventh year and are here inserted at the end of the eighth year, so as to take advantage of the increased maturity and experience of the pupils. An effort has been made to use in the business arithmetic as much as possible of the pupil's knowledge of algebra. The choice of materials has been determined by the demands of the everyday life of the average citizen rather than by an attempt to cover a large number of topics, many of which will never come within the experience of most people.

The book closes with extended groups of problems which apply the mathematical principles previously taught to conditions under which such problems are usually met. The last few sets of problems are involved in projects planned to follow as closely as possible a series of activities requiring much computation for a genuine purpose. These projects may suggest to the teacher similar projects having particular local interest for the pupils.

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JUNIOR HIGH SCHOOL MATHEMATICS

SECOND BOOK

CHAPTER I

REPRESENTING NUMBERS BY LETTERS. FORMULAS

1. The use of letters as numbers. In the First Book it has been shown that the use of letters to represent numbers gives a kind of shorthand in which rules may be written and remembered, and that many problems can be solved by substituting for the letters the numbers which they represent.

But these formulas may be used in other ways to solve much more difficult problems and to discover new rules. In order so to use them, however, it is necessary to acquire some skill in handling letters representing numbers. The pupil must learn to use these new symbols in adding, subtracting, multiplying, and dividing numbers ; he must also learn how to interpret a problem expressed in these symbols.

2. The use of letters to shorten statements. The pupil has learned that

The sum of two numbers x and y is written $x+y$.

The difference when y is subtracted from x is written $x-y$.

The product of x and y is written xy, or $x \cdot y$, or $x \times y$.

The quotient of x divided by y, is written $\frac{x}{y}$ or $x \div y$.

The pupil must acquire skill in interpreting these forms and in using them with both letters and figures.

Exercise 1

1. A boy has m marbles and finds 10 more. How many has he then?

2. In an orchard of n trees 17 trees die. How many trees are then alive?

3. In a class there are x boys and y girls. How many pupils are there in the class? How many more boys than girls?

4. What is the perimeter of a triangle if the sides are a, a, and b? If the sides are a, a, and a? If the sides are $2x$, $3k$, and 16?

5. What number is 35 more than x? What number is 67 less than $2y$?

6. What is the cost of y yards of cloth at n cents a yard?

7. How many dollars in c cents?

8. It costs d dollars a hundred to get some bills printed. How much does each bill cost?

9. What is the next integer after 8? After 14? What must be done to an integer to get the next integer following it?

10. If a is an integer, what is the next integer after it? What is the next integer before it?

11. How far does a train go in t hours at the average rate of m miles an hour?

12. If a man walks x miles in 5 hours what is his rate per hour? If he walks y miles in h hours what is his rate per hour?

13. What number is 8 greater than 3 times x?

14. A has x dollars, B has twice as much, and C has \$36 more than B. How much has B? How much has C?

15. How many inches in f ft.? In f ft. 6 in.?

16. What number is $\frac{1}{2}$ of n more than n?

17. What number is $\frac{4}{5}$ of m less than m?

18. What number is 3 times h more than k?

19. The numbers 1, 2, 3, 4, 5 are consecutive integers. Name four consecutive integers beginning with 8. What must be added to an integer to get the next consecutive integer? Suppose that n is an integer. Name the next four consecutive integers.

20. A man who is 11 miles from home walks toward home at the rate of 3 miles an hour. How far from home is he at the end of 3 hours?

21. A and B are traveling in automobiles in the same direction. A is 15 miles behind B, and is traveling 6 miles an hour faster than B. How long before A will overtake B?

22. A man who is m miles from home walks toward home at the rate of 3 miles an hour. How far from home is he at the end of t hours? How far from home would he have been if he had walked away from home instead of toward it?

23. In exercise 21 suppose that A is k miles behind B and travels r miles an hour faster. How long before A will overtake B?

24. While the hour hand passes over 5 minute spaces how far does the minute hand go? The minute hand goes how many times as fast as the hour hand?

25. While the hour hand passes over h minute spaces how far does the minute hand go?

26. A boy is now m years old. How old was he 4 years ago?

27. A man is y years old. How old will he be in r years?

28. What is the cost of y yards, f feet, 6 inches of ribbon at 3 cents a foot?

29. A string is just long enough to wind 6 times around a wheel whose radius is r inches and have i inches over. How long is the string?

30. How much longer is the circumference of a circle of radius r than one of radius 2?

3. Statement of rules as formulas. We have seen that it is often very convenient to write rules as formulas. The formula is shorter, more easily remembered, and easier to use in calculating. It is important to know

(*a*) How to translate a rule or other statement in words into a formula.

(*b*) How to translate a formula into an English sentence.

(*c*) How to use a formula in calculating.

We shall now have practice in dealing with formulas in the first of these ways, and later in each of the other two ways.

EXAMPLE. If a body is let fall from the top of a tower the number of feet, s, that it falls in a certain number of seconds, t, is 16.1 times the square of the number of seconds. This principle is briefly stated in the formula

$$s = 16.1\, t^2.$$

Exercise 2

Write the formulas which give the answers to the following exercises :

1. Find the selling price, s, when the cost is c and the gain is g.

2. Find the average, m, of six numbers a, b, c, d, e, and f.

3. Using the same letters that you used in percentage problems, find (*a*) the percentage, given the rate and the base ; (*b*) the base, given the rate and the percentage ; (*c*) the rate, given the base and the percentage.

4. Write a formula for finding

(*a*) The quotient, q, given the dividend, D, and the divisor, d.

(*b*) The dividend, D, given the divisor, d, the quotient, q, and the remainder, r.

(*c*) One factor, F, when the other factor, f, and the product, p, are given.

5. Write a formula for finding

(*a*) The number of feet, f, of bookshelves in a room which contains a shelves each n feet long, and 4 shelves each 6 feet long.

(*b*) The number of seats, s, in a room which contains x double desks (desks that will seat 2 persons), a single desks, and 1 chair.

(*c*) The number of bushels of corn, n, that a farmer raises, if he gets 40 bushels an acre from x acres, and 60 bushels an acre from y acres.

Write formulas for the answers to the following exercises:

6. A tank is being filled by two pipes. It now contains 400 gallons. How many gallons will it contain after 45 minutes if one pipe brings in a gallons a minute, and the other b gallons a minute?

7. In a certain year the farmers of Iowa raised a acres of corn which gave an average yield of b bushels per acre. The next year they raised A acres and the yield increased to B bushels per acre. The total increase in the yield was I bushels. Find I.

8. There were 200 pounds of flour in a barrel. There have been used from it p pounds a day for 10 days, and 5 pounds a day for d days. How much remains? Let b represent the number of pounds that remain.

9. The weight of 40 eggs and a basket is x ounces. The basket weighs y ounces. How many ounces do the eggs weigh? Find the weight, w, of one egg.

10. A board is n feet long. From it are cut 7 pieces of equal length, leaving a piece 2 feet long. What is the sum of the lengths of the 7 equal pieces? Find the length, l, of one of the equal pieces.

11. A boy now has \$15.35. He makes \$1.25 a day and spends \$1.05. What is the sum of money, s, that he has at the end of d days?

12. A boy solves n problems on Tuesday and 20% more on Wednesday. Find the number, N, that he solves on Wednesday.

13. Let x be the total number that he solves on both Tuesday and Wednesday. Find x.

14. The cost of an article is c. The selling price is 10% less than the cost. Find the selling price.

15. A bank lends \$3000 at n% interest. Find the interest, i, for 1 year ; for x years.

16. The cost of an article is c dollars. It is marked to sell at 25% above the cost. The article becomes damaged and is sold for \$2 less than the marked price. The selling price is s dollars. Find s.

17. A bookshelf will hold x books 1 inch thick, and y books $1\frac{1}{2}$ inches thick. The shelf is l feet long. Find l.

18. General admission to a baseball game is 50 cents, reserved seats 75 cents. There are x general admissions sold, and y reserved seats. Find the total receipts, r.

19. The rule for making coffee in camp is " One spoonful for each person and one for the pot." Write this rule as a formula where n is the number of spoonfuls and p is the number of persons.

20. A cistern contains 125 barrels of water. One pipe carries water into it at the rate of m gallons an hour, and another takes water out at the rate of n gallons an hour. After h hours there are g gallons of water in the cistern. Find g.

21. Find Mr. Blank's income, I, if he has invested a dollars at 5%, b dollars at 6%, and c dollars at 7%, and has a salary of d dollars.

22. A string whose length is l is 3 inches too short to go around a rectangle $a'' \times b''$. Find l.

4. Translating formulas into English sentences. To understand and explain a formula it is often necessary to be able to translate it into an English sentence.

EXAMPLE. We have had the formula, $D=dq+r$, for finding the dividend when the divisor, quotient, and remainder are given. This formula may be stated: The dividend equals the product of the divisor and quotient plus the remainder.

Exercise 3

State in words the rule expressed by each of the following formulas :

1. $A=ab$. The formula for the area of a parallelogram.
2. $A=\frac{ab}{2}$. The formula for the area of a triangle.
3. $A=\frac{a(b+b')}{2}$. The formula for the area of a trapezoid.
4. $A=\pi r^2$. The formula for the area of a circle.
5. $A=s^2$. The formula for the area of a square.
6. $p=rb$. This and the next two are percentage formulas.
7. $b=\frac{p}{r}$. 8. $r=\frac{p}{b}$.
9. $i=prt$. This and the next three are interest formulas.
10. $p=\frac{i}{rt}$. 11. $r=\frac{i}{pt}$. 12. $t=\frac{i}{pr}$.
13. $V=lwt$. The formula for finding the volume of a rectangular solid.
14. $V=e^3$. The formula for finding the volume of a cube.
15. $d=rt$. The formula for finding the distance when the rate and time are given.
16. $g=rc$. Gain, rate, and cost formula.
17. $s=c+rc$. The formula for finding the selling price, given the cost and the rate of gain.

5. Substitution in formulas. The engineer and mechanic make much use of the formula in calculation. In order to use the formula it is necessary to be able to substitute numbers for the letters and to simplify the results.

EXAMPLE. A bomb is dropped from an airplane. How far will it fall in 20 seconds?

The answer to this question may be found by using the formula given on page 4, $s=16.1t^2$. Substituting in this formula we have

$s=16.1\times 20^2=6440$, the number of feet the bomb falls in 20 seconds.

Exercise 4

1. Use the formula for finding the volume, V, of a rectangular solid when the length, l, the width, w, and the thickness, t, are given, and find V when l, w, and t have the following values. Copy and fill out this table.

l	16	36	400	12.8	$5\frac{1}{3}$
w	8	2.4	87	3.4	$2\frac{1}{2}$
t	.3	.9	33	2.5	.4
V					

2. The product $8\times 8\times 8$ is written 8^3, which is read "8 cube." The product $n\times n\times n$ is written n^3, which is read "n cube." The formula $V=e^3$ gives the volume of a cube whose edge is e. Find V for the following values of e: 6 ; 12 ; 25 ; .7 ; 8.6 ; 7.02 ; $\frac{7}{8}$; $3\frac{1}{4}$.

3. Show that the area of Figure 1 is given by the formula $A=a^2-b^2$. Find A when a and b have the following values. Copy and fill out this table.

a	12	5.6	342	$\frac{5}{8}$	$4\frac{1}{4}$	.42
b	3	1.5	90	$\frac{1}{4}$	$1\frac{1}{2}$	.09
A						

4. Show that the area of Figure 2 is given by the formula $A = a^2 - 4b^2$. In making tinplates of this form how many square feet of tin are required to make 1000 of each of the following sizes, no allowance being made for waste? Copy and fill out this table.

a	10 in.	8 in.	16 in.	$6\frac{1}{4}$ in.
b	2 in.	$1\frac{1}{2}$ in.	$3\frac{1}{2}$ in.	$1\frac{1}{8}$ in.
A				
1000 A				

5. Make a formula for finding the perimeter of Figure 2. Find the perimeter for the values of a and b given in the preceding exercise.

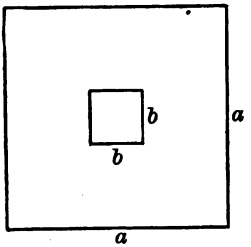


FIG. 1

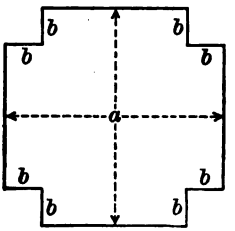


FIG. 2

6. Use the formula $s = 16.1t^2$ for finding the number of feet that a body starting from rest falls in 3 sec.; in 15 sec.; in 30 sec.

7. Show that a body starting from rest will fall nearly one mile in 18 sec.

8. Use the percentage formulas for finding the missing numbers in the following table:

b	\$6480	5280 ft.	24 hr.		\$25,000	
r	6%			5%	.15%	.3%
p		40 rd.	72 hr.	\$8.96		$7\frac{1}{2}$

9. Use the interest formulas for finding the missing numbers in the following table:

p	\$5000	\$4000	\$800	
r	4%	5%		7%
t	2 yr.		90 da.	60 da.
i		\$700	\$12	\$14.70

10. Find the value of $3x+6$ when $x=4$; when $x=3.5$.

11. Find the value of $5x+2$ when $x=98$; when $x=34.1$; when $x=11$.

12. Find the value of $4x-3$ when x has each of the following values: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

13. Find the value of $6n+1$ when n has each of the following values: 0, .2, .4, .6, .8, 1, 1.2.

14. Find the value of $2x^2-x+2$ when x has each of the following values: 0, .1, .5, 1, 1.5, 2.

15. Find the value of $3a^2+a-4$ when a has each of the following values: 1, 1.1, 1.6, 2.

16. Find the area of a trapezoid whose altitude is 20 in., and bases 36 in. and 40 in.

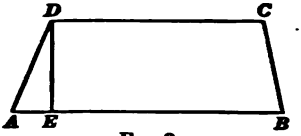


FIG. 3

17. How many acres in a field in the form of Figure 3, if $AB=36$ rods, $CD=28$ rods, and $DE=12$ rods?

18. Find the area of this plate, Figure 4, if the radius of the plate is 2 ft. 6 in. and the radius of each of the holes is 4 in.

19. The area of a circle of radius 12 in. is how many times the area of a circle of radius 4 in.? Estimate the answer and then compute it.

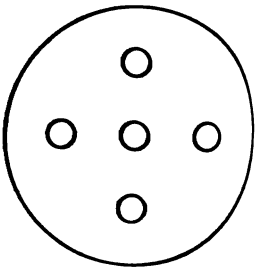


FIG. 4

20. In the formula $d=rt$, d represents the distance traveled by a moving object, r the rate of motion, and t the time. Use this formula to find

(*a*) The distance a train travels in 18 hr. at the rate of 27 mi. per hour.

(*b*) The number of hours it takes an airplane to travel 640 mi. at the rate of 75 mi. per hour.

(*c*) The rate a bird flies if it flies 720 mi. in $12\frac{1}{2}$ hr.

21. Make a formula that states that p is the product of the factors f and f'. How can you find f if you know p and f'? How can you find f' if you know p and f?

22. Find f if $p=954$ and $f'=24$.

23. Find f' if $f=.3$ and $p=\$65$.

24. Find f if $f'=15\%$ and $p=46.5$.

25. The load that may be safely attached to an iron chain is given by the formula, $L=7.11d^2$, where L is the load in tons, and d is the diameter of the chain iron in inches. Find the safe load when the chain iron has the following diameters: $\frac{1}{4}$ in.; $\frac{1}{2}$ in.; 1.3 in.

26. The number of pounds of blasting powder, p, used to blow a hole in a wall of thickness t feet is given by the formula $p=\frac{t^3}{80}$. How many pounds must be used to blow a hole through a wall 3 ft. thick? 4 ft. thick? 6 ft. thick?

27. The total surface, T, of a cylinder is found by the formula $T=2\pi r\,(r+h)$, in which r is the radius of the cylinder and h is its altitude. Find the total surface of a cylinder whose radius is 3 in. and height 7 in. Find the total surface of a cylinder whose radius is $\frac{3}{4}$ ft. and height .7 ft.

28. The amount, A, of a certain principal, p, in t years at rate, r, is given by the formula $A=p(1+rt)$. Find A when p is \$450, r is 5%, and t is 2 yr. 3 mo.; also find A when $p=\$5460$, $r=7\%$, and $t=1$ yr. 4 mo.

29. The beam in Figure 5 is supported at one end and supports a load at the other end. The heaviest load that a steel beam in such a position will support without breaking is given by the formula $W = \frac{6800\,bd^2}{l}$ in which W is the load in pounds, b the width, d the depth or thickness, and l the length of the beam, all of these dimensions being in inches.

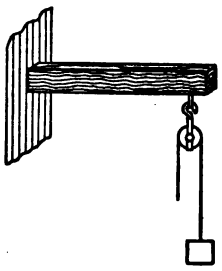


FIG. 5

Find the heaviest load that can be supported at the end of a steel beam (*a*) 6 ft. long, 2 in. wide, and 3 in. deep. (*b*) At the end of one 8 ft. long, and 3 in. square on the end.

30. The horsepower of certain types of gasoline automobile engines is given by the formula $\text{H.P.} = \frac{ND^2}{2.5}$, where H. P. is the number of horsepower, N is the number of cylinders, and D is the diameter of the cylinders. Find the number of horsepower of the engine in each of the following kinds of automobiles :

NAME OF CAR	N	D
Buick	6	$3\frac{1}{4}$ in.
Dodge	4	$3\frac{7}{8}$ in.
Ford	4	$3\frac{3}{4}$ in.
Cadillac	8	$3\frac{1}{8}$ in.
Packard	6	$4\frac{1}{2}$ in.
Packard	12	3 in.
Pierce-Arrow	6	5 in.

CHAPTER II

FUNDAMENTAL OPERATIONS AND EQUATIONS

6. Addition and subtraction. Such sums as $7 \cdot 8+3 \cdot 8$ are usually found in arithmetic by first multiplying and then adding. Thus,

$$7 \cdot 8+3 \cdot 8=56+24=80.$$

Since these two terms contain a common factor the sum may be found by first adding and then multiplying. Thus,

$$7 \cdot 8+3 \cdot 8=10 \cdot 8=80.$$

When terms to be added have a common factor the second way is generally used. Thus in finding the sum, $7x+3x=10x$, the 7 and 3 are first added and x is multiplied by their sum.

What has been said about sums applies also to finding the difference of two numbers. Thus $7 \cdot 8-3 \cdot 8=4 \cdot 8=32$, and $7x-3x=4x$.

Since $ax+bx$ means x taken a times added to x taken b times, the sum is x taken $a+b$ times. This is written,

$$ax+bx=(a+b)x.$$

Exercise 5

Find the following sums and differences. First add or subtract, then multiply.

1. $5 \cdot 7+8 \cdot 7$.
2. $25 \cdot 6+19 \cdot 6$.
3. $12 \cdot 9+4 \cdot 9$.
4. $15 \cdot 7+3 \cdot 7$.
5. $15 \cdot 3-7 \cdot 3$.
6. $37 \cdot 7-27 \cdot 7$.
7. $84 \cdot 17-64 \cdot 17$.
8. $246 \cdot 359-239 \cdot 359$.

9. $87 \cdot 463 + 13 \cdot 463$.
10. $4 \cdot 5 + 4 \cdot 7$.
11. $13 \cdot 8 + 13 \cdot 12$.
12. $7 \cdot 3 - 3 \cdot 5$.
13. $9 \cdot 4 + 4 \cdot 6$.
14. $29 \cdot 34 + 34 \cdot 11$.
15. $57 \cdot 29 - 17 \cdot 29$.
16. $\frac{1}{3} \cdot 7 + \frac{2}{3} \cdot 7$.
17. $\frac{1}{3} \cdot 17 + \frac{1}{3} \cdot 17$.
18. $3\frac{5}{6} \cdot 35 - 2\frac{1}{3} \cdot 35$.
19. $.85 \cdot 46 + .05 \cdot 46$.
20. $.1 \cdot 85 - .09 \cdot 85$.

21. Check each of the preceding exercises by performing the multiplication first, then the addition or subtraction. Which is the shorter way?

Find the following sums and differences :

22. $9x + 7x$.
23. $3a + 17a$.
24. $5mn + 12mn$.
25. $8c + c$.
26. $r + 15r$.
27. $14b - 6b$.
28. $18n - 2n$.
29. $5x - x$.
30. $19y - y$.
31. $12a - 11a$.
32. $7c + 5c - 8c$.
33. $18mn + 13mn - 12mn$.
34. $9a^2 - 5a^2 - 3a^2$.
35. $\frac{1}{2}ab + \frac{3}{4}ab + \frac{5}{6}ab - \frac{1}{6}ab$.
36. $2 \cdot 17 + 17 \cdot 5 - 3 \cdot 17$.
37. $8 \cdot 42 + 3 \cdot 42 - 42 \cdot 4$.
38. $.7n + 1.6n + 3.05n$.
39. $\frac{1}{2}x + x$.
40. $4r - r$.
41. $\frac{1}{2}x + \frac{3}{8}x + x$.
42. $9 \cdot 13 + 13 \cdot 17$.
43. $a + \frac{2}{3}a - \frac{5}{6}a$.
44. $45b + 18b + b - 10b$.
45. $3.8a - 2.1a + .04a$.
46. $.0\frac{1}{2} \cdot 7 + .05 \cdot 7$.

47. $rx + sx$.
48. $na + ba$.
49. $ax + ay$.
50. $rb + cr$.
51. $xa + 2a$.
52. $5r + 2r$.
53. $6n + bn$.
54. $7a + a$.
55. $na + a$.
56. $ay - by$.
57. $cn - xn$.
58. $ar - na$.
59. $ay - 2y$.
60. $9m + mx$.
61. $ad - ca$.
62. $y + ry$.
63. $ry + r$.
64. $c + rc$.
65. $c - rc$.
66. $p + prt$.
67. $bh + b'h$.
68. $r - .1r - .01r$.
69. $2mp - 3p$.
70. $ac - bc + nc$.
71. $rx + xy - xz$.
72. $ap - bp + p$.

In expressions such as $3a + 5 + 7a + 8$ the terms do not all contain a common factor. Such expressions are simplified by combining the terms which have a common factor. Thus,

$$3a + 5 + 7a + 8 = 10a + 13.$$

Exercise 6

Combine terms as far as possible:

1. $2x+8x+8$.
2. $9a+6+10$.
3. $12d+8-4$.
4. $7y+5+2y+20$.
5. $14b+21+b-8$.
6. $8+4n-3+7n$.
7. $8x+4-4$.
8. $12c-5c-3c$.
9. $9y+35-9y$.
10. $24a+6-a-5$.
11. $a+1+a+1$.
12. $8a-7a+18+30$.
13. $3a-a+18-17$.
14. $7y-4y-2y+12y$.
15. $7n-7+n$.
16. $8a-a$.
17. $6x-4x-2y+12x$.
18. $9y+3y+8x-x$.
19. $5m+15a-5m-12a$.
20. $\frac{1}{2}n+n+3$.
21. $\frac{2}{3}n-4-\frac{1}{2}n+6$.
22. $\frac{5}{8}c-\frac{1}{4}+2c-\frac{1}{8}c$.
23. $.4x+.206+2.2x-.08$.
24. $ax+by+cx-dy$.

The expression $ax+bcd+a^2t$ is composed of three *terms*. The first term is ax, the second is bcd, and the third is a^2t.

If a term is separated into two factors either of the two factors may be called the **coefficient** of the other. Thus the term bcd may be separated into the two factors b and cd. Then b is the coefficient of cd, and cd is the coefficient of b.

Like terms are terms having a common factor. Thus $3x$ and $5x$ are like terms; also ax and bx.

Rule. *To add like terms, multiply their common factor by the sum of its coefficients.*

EXAMPLE 1. Add $4 \cdot 7+7 \cdot 6+7$.

SOLUTION. The common factor of these three terms is 7. Its coefficient in the first term is 4, in the second term is 6, and in the third term is 1. The sum of the coefficients is 11.

Then
$$4 \cdot 7+7 \cdot 6+7=(4+6+1)7$$
$$=11 \cdot 7.$$

EXAMPLE 2. Add $ax+3x+xy$.

SOLUTION. $ax+3x+xy=(a+3+y)x$. What is the common factor? What is its coefficient in each term?

Exercise 7

1. Read the terms of the expression $4x^2+7xy+3yx$. What is the coefficient of x^2 in the first term? Of xy in the second term? Of x in the third term? Of x in the second term?

2. In the expression $2x+3x$ what are the factors of the first term? Of the second term? What is the common factor of the two terms? What is its coefficient in the first term? In the second term? Find the sum of the coefficients. Multiply this sum by the common factor.

3. Answer the questions of the preceding exercise for the expression $xa+cx$.

Combine the terms in the following :

4. $ab+bc$.
5. $5s+7s$.
6. $pr+pt$.
7. $a+ac$.
8. $2hs+3ns$.
9. $5xy-4ry$.
10. $7ay+4ab-3ac$.
11. $s+2s+sx$.
12. $a^2+2a^2+5a^2-7a^2$.
13. $5\cdot 7\cdot 3+4\cdot 7\cdot 3$.
14. $6\cdot 9\cdot 4+2\cdot 9\cdot 5$.
15. a^2+ra.
16. $2ab-a^2$.
17. $3cn-5cd+2c$
18. $7a^2b+2ab^2$.
19. n^2m+2n^2r.
20. $\frac{1}{2}t+\frac{3}{4}t$.
21. $1.1ab+.25ab+a+3a$.
22. $.01at^2+.1at^2-.02at^2$.
23. $2r^2+\pi r^2-.5r^2$.

7. Equations. The statement that two expressions are equal is called an **equation.** The two expressions are called the **members** of the equation. Thus in the equation $3x+6=2x+8$, $3x+6$ is the left member and $2x+8$ is the right member.

The formulas that we have used are examples of equations.

8. Questions asked by equations. Many questions of elementary arithmetic may be stated in the form of equations. Thus the question, What number added to 4 gives 11? may be written, $4+?=11$, or $4+n=11$, where n stands for the unknown number.

Exercise 8

State the following as equations and give the answers :

1. What number added to 9 gives 16?
2. What number subtracted from 19 gives 6?
3. Forty-four is 8 more than what number?
4. Six times what number is 45?
5. One-half of what number is 19?
6. Thirteen is one-eighth of what number?
7. What must be subtracted from 42 to get $12\frac{1}{2}$?
8. What must 8 be multiplied by to get 92?

Ask questions which are equivalent to the following and give the answers :

9. $?+4=12$.	11. $18-?=0$.	13. $\frac{1}{5}$ of $?=13$.
10. $6+?=26$.	12. $\frac{2}{3}$ of $?=8$.	14. $?+34=65$.

In the following the number to be found is represented by x. Find x.

15. $x+3=13$.	19. $7x=21$.	23. $3x=150$.
16. $x-13=2$.	20. $9x=45$.	24. $\frac{1}{7}x=21$.
17. $27+x=35$.	21. $2x=7$.	25. $\frac{2}{5}x=4$.
18. $45-x=23$.	22. $\frac{1}{3}x=10$.	26. $12\div x=12$.

Many of the equations with which the pupil will have to deal should be thought of as questions. The equation, $2n+1=7$, asks the question, For what value of n is $2n+1$ equal to 7? The answer is 3. We test the truth of this answer by substituting 3 for n in the equation. This gives $6+1=7$, or $7=7$, which shows that the answer is correct.

This testing the answer by substituting is called **checking**.

If, in this example, we say that the answer is 4 instead of 3, and then substitute 4 for n in the equation, we get $8+1=7$, or $9=7$, which is not true.

The number 3 is said to **satisfy** the equation $2n+1=7$.

A number that satisfies an equation is called a **root** of the equation.

The process of finding the root of an equation is called **solving** the equation.

We shall learn how to solve certain kinds of equations.

The letter in an equation whose value we wish to find is called the **unknown**. In the equation $2n+1=7$, n is the unknown.

When an equation is solved for an unknown the unknown will stand by itself in one member of the equation, and its value will be given by the other member of the equation. Thus, when the equation $2n+1=7$ is solved for n, we have the equation $n=3$. The pupil should note that when an equation is solved for an unknown, *this unknown is found in only one member.*

9. The equation as a balance. The equation may be thought of as a balance. Thus, in the equation $x+2=14$, $x+2$ and 14 may be thought of as weights which balance when $x+2$ is placed in one scale pan and 14 in the other. (See Figure 6.)

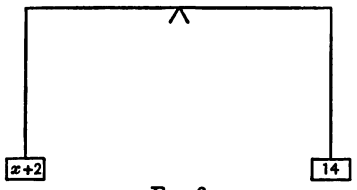


FIG. 6

When one set of weights balances another set, if a weight is added to or taken from one scale pan, a weight of equal size must be added to or taken from the other pan if the resulting weights are still to balance. Thus, if we subtract 2 from $x+2$ we must subtract 2 from 14 if we wish the resulting weights to balance each other. This means that $x=12$.

Suppose that $3x$ pounds balance 15 pounds, that is, $3x=15$. Then $\frac{1}{3}$ of $3x$ pounds will balance $\frac{1}{3}$ of 15 pounds. That is, x pounds will balance 5 pounds, or $x=5$.

Suppose that $\frac{1}{2}x$ pounds balance 7 pounds, that is, $\frac{1}{2}x=7$. Then 2 times $\frac{1}{2}x$ pounds will balance 2×7 pounds. That is, x pounds will balance 14 pounds, or $x=14$.

Exercise 9

1. Suppose that the left pan of a balance contains $x+5$ pounds and the right pan 15 pounds. Let the 5 pounds be taken from the left pan. Then what must be done to the right pan to maintain a balance? Then x equals how many pounds?

2. The left pan contains $x-12$ pounds and the right 7 pounds. Add 12 pounds to the left pan. Then what must be done to maintain a balance? Then x equals how many pounds?

3. If $3x$ pounds just balance 12 pounds, x pounds will balance how many pounds? What must be done with $3x$ to get x? What must be done with 12 to get the number that balances x?

4. Answer similar questions if $5x$ pounds balance 20 pounds, that is, if $5x=20$.

5. Answer similar questions if $\frac{1}{3}x$ pounds balance 12 pounds, that is, if $\frac{1}{3}x=12$; if $\frac{1}{2}x=8$; if $7x=21$.

6. If the weights in the two scale pans balance, tell what must be done to the weights in the other pan

(*a*) If a certain number of pounds is added to the weight in one pan.

(*b*) If a certain number of pounds is subtracted from the weight in one pan.

(*c*) If the weight in one pan is divided by a certain number.

(*d*) If the weight in one pan is multiplied by a certain number.

7. If we wish to keep the two members of an equation equal, what must be done to one member

(*a*) If a certain number is added to the other?

(*b*) If a certain number is subtracted from the other?

(*c*) If the other is multiplied by a certain number?

(*d*) If the other is divided by a certain number?

10. Principles concerning equal numbers. In solving equations much use is made of certain principles concerning equal numbers.

Exercise 10

1. In a certain room there are d seats in desks and 5 chairs, in which to seat m men and 10 women. What is the total number of seats? What is the total number of persons?

2. Suppose that the number of seats is the same as the number of persons. State this fact as an equation.

3. If three more persons enter and three seats are brought in, how many persons are there? How many seats? How does the number of persons compare with the number of seats? State this as an equation.

4. If now seven persons leave and seven seats are taken out, how many seats and how many persons remain? How do these numbers compare? State this fact in symbols.

5. If a and $b+2$ are equal numbers and 3 is added to each of them, what are the results? Which is the greater? State this fact in symbols.

6. If n is added to each of two equal numbers what is true of the sums?

7. If k is subtracted from each of two equal numbers what is true of the remainders?

8. I am thinking of two equal numbers. I add the same number to each of them. What is true of the sums?

9. I am thinking of two equal numbers. I subtract the same number from each of them. What is true of the remainders?

10. Suppose that a and b are equal numbers. Multiply each by 6. What are the products? How do they compare in value? State this fact in symbols.

11. If each of two equal numbers is multiplied by the same number what is true of the products?

12. If each of two equal numbers is divided by the same number what is true of the quotients?

It has been illustrated in the above exercises that if we start with equal numbers, we shall obtain equal results if we

add to them the same or equal numbers;

subtract from them the same or equal numbers;

multiply them by the same or equal numbers;

divide them by the same or equal numbers.

These truths are usually stated in the following form:

Principle I. *Equal numbers added to equal numbers give equal sums.*

Principle II. *Equal numbers subtracted from equal numbers give equal remainders.*

Principle III. *Equal numbers multiplied by equal numbers give equal products.*

Principle IV. *Equal numbers divided by equal numbers give equal quotients.*

We shall find much use for these principles in what follows.

Exercise 11

1. Which of the above principles is illustrated in exercise 5, page 20? In exercise 7? In exercise 10? In exercise 12?

Give the reason for the conclusion in each of the following:

2. $a=4$, $b=5$; then $a+b=9$.
3. $a=2$; then $6a=12$.
4. $3a=15$; then $a=5$.
5. $x=12$, $y=9$; then $x-y=3$.
6. $\frac{n}{6}=10$; then $n=60$.
7. $r=4$, $s=6$; then $rs=24$.
8. $az=r$; then $z=\frac{r}{a}$.
9. $2m+b=5$; then $2m=5-b$.
10. $r+s=t$; then $r=t-s$.

11. Operations used in solving equations. The principles just given are used in the operations performed in solving equations. The pupils must now learn which of the operations to perform in solving a given equation. The operations to be used are those which, in the given equation, will result in leaving the unknown standing alone as one member of the equation.

Exercise 12

1. What number is 6 greater than x? 7 greater than y? 9 greater than $2x$?

2. Which is the greater b or $b+2$? How much? Answer similar questions about a and $a+5$; $2x$ and $2x+8$; $5m+75$ and $5m$; $8r+45$ and $8r$.

3. What number is 4 less than m? 8 less than x? 16 less than $3y$? 68 less than $5x$?

4. Which is the greater $a-2$ or a? How much? Answer similar questions about $r-3$ and r ; $y-47$ and y ; $3v$ and $3v-21$; $6z$ and $6z-80$.

5. What must be added to or subtracted from:
(a) $x-12$ to get x? (b) $x+12$ to get x? (c) $3x+7$ to get $3x$? (d) $9y-53$ to get $9y$?

6. What must be done to $a+32$ to get a? To $r-14$ to get r? To $8t-27$ to get $8t$? To $31b+965$ to get $31b$?

7. Suppose that $a+1=6$. What must be subtracted from $a+1$ to get a as a result? If the same number is subtracted from 6 what is then true of the two results? What is a then equal to? Which of the above principles may be used here?

8. Ask and answer questions similar to those in the previous exercise if $a+7=12$.

9. If $n+5=20$, what can be done to both members of the equation to find the number that n equals? Answer a similar question if $y+14=36$.

10. Suppose that $x-5=7$. What must be added to $x-5$ to get x as a result? If the same number is added to 7 what is true of the two results? Hence what number is x equal to? Which of the above principles have you used?

11. Ask questions similar to those in the previous exercise if $x-9=17$.

12. If $y-3=8$, what can be done to both members of the equation to find the number that y equals? Answer a similar question if $y-10=22$. What principle applies?

13. Tell what each of the following must be divided by to get x: $12x$; $9x$; $70x$; $956x$; ax; $8.6x$; $4.5x$; $.6x$; $2\frac{1}{2}x$; $3\frac{1}{3}x$; $\frac{3}{4}x$; $\frac{8}{15}x$.

14. In the equation $2n=10$, by what must $2n$ be divided to get n as a quotient? Then n equals what number? Which of the above principles is used here?

15. Ask questions similar to those in the last exercise if $3n=24$; if $7n=14$.

16. The two numbers $3x$ and 30 are equal. Each is divided by 3. What are the quotients? How do the values of the quotients compare? Then x equals what number?

17. If you know that $7a=56$, how can you find the number that a equals? What principle is used?

18. By what must each of the following be multiplied to get y: $\frac{1}{2}y$; $\frac{1}{3}y$; $\frac{1}{45}y$; $\frac{1}{100}y$; $.1y$; $.01y$?

19. In the equation $\frac{1}{3}x=6$, what must $\frac{1}{3}x$ be multiplied by to get x as a product? Then x equals what number? What principle is used?

20. Answer questions similar to those in the last exercise if $\frac{1}{4}x=7$; if $\frac{1}{6}x=20$.

21. The numbers $\frac{1}{6}n$ and 2 are equal. Each is multiplied by 6. How do the values of the products compare? Then n equals what number? What principle is used?

22. If you know that $\frac{1}{7}y=14$, how can you find what number y equals? What principle is used?

23. What is meant by solving an equation?

24. To solve the equation $x+7=10$, what is done to each member? Answer the same question for the equation $n-9=6$; for the equation $5x=45$; for the equation $\frac{1}{3}y=4$.

25. Give an equation that may be solved by adding the same number to both members; one that can be solved by subtracting the same number from both members.

26. Give an equation that can be solved by multiplying both members by the same number; one that can be solved by dividing both members by the same number.

Exercise 13

State what must be done to find x, and find it, in the following. State which principle is used.

1. $x+4=19$.

SOLUTION. Subtract 4 from each member of the equation. Principle 2. Then $x=15$. Check. $15+4=19$.

2. $x-9=11$.

SOLUTION. Add 9 to each member of the equation. Principle I. Then $x=20$. Check. $20-9=11$.

3. $\frac{1}{7}x=8$.

SOLUTION. Multiply each member by 7. Principle III. Then $x=56$. Check. $\frac{1}{7}$ of $56=8$.

4. $2\frac{2}{5}x=57$.

SOLUTION. Divide each member of the equation by $2\frac{2}{5}$. Principle IV. Then $x=24$. Check $2\frac{2}{5}\times 24=57$.

5. $x-19=13$.

6. $12+x=27$.

7. $x-15=11$.

8. $5x=35$.

9. $3.4x=238$.

10. $.06x=15$.

11. $\frac{2}{3}x=8$.

12. $\frac{1}{6}x=11$.

13. $.1x=10$.

14. $100x=.01$.

15. $x-2=0$.

16. $5x=0$.

17. $\frac{1}{12}x=1$.

18. $250x=20$.

19. $17+x=17$.

20. $.3x=.01$.

12. Solving equations. In the following solutions you will understand how to go from one equation to the next if you can answer the following questions :

(*a*) What is done?

(*b*) Why is it done?

(*c*) Why are the results equal?

EXAMPLE 1. Solve for n, $4n+9=21$.

SOLUTION.

(1) $4n+9=21.$
(2) $4n=12.$
(3) $n=3.$

The answers to the above questions when going from (1) to (2) are :

(*a*) 9 is subtracted from each member of the equation.

(*b*) So that the terms not involving n shall not be found in the left member.

(*c*) Equal numbers subtracted from equal numbers give equal remainders.

The answers when going from (2) to (3) are :

(*a*) Each member of the equation is divided by 4.

(*b*) So that n shall stand alone in one member.

(*c*) Equal numbers divided by equal numbers give equal quotients.

In the following solution the answers to questions (*a*) and (*c*) only are given. The pupils should be able to answer question (*b*).

EXAMPLE 2. $7x-5=27+3x.$

SOLUTION.

$7x-5=27+3x.$
$7x=32+3x.$ Adding 5 to each member. Principle I.
$4x=32.$ Subtracting $3x$ from each member. Principle II.
$x=8.$ Dividing each member by 4. Principle IV.

CHECK. $56-5=27+24.$
$51=51.$

Exercise 14

Solve each of the following equations for the letter involved. Be prepared to explain the solution as in example 1. Check each answer.

1. $x+9=20$.
2. $y+12=32$.
3. $x+43=86$.
4. $n-12=5$.
5. $a-9=9$.
6. $a+9=9$.
7. $20+x=42$.
8. $5n=40$.
9. $6y=20$.
10. $35z=315$.
11. $10b=6$.
12. $5a+7=52$.
13. $8y+14=122$.
14. $9x-35=109$.
15. $2a=a+12$.
16. $12a=5a+91$.
17. $24r=12r+6$.
18. $r=9-r$.
19. $8z=12-2z$.
20. $9x-3x=81$.
21. $4x+9x=110-19$.
22. $40=35+x$.
23. $24+90=3x$.
24. $5n-7=83$.
25. $4n=33+n$.
26. $5n-7=33+n$.
27. $3x+9=x+31$.
28. $12y+14=3y+77$.
29. $5a+4=4+a$.
30. $7r+6=44+3r$.
31. $8a=16$.
32. $.8a=16$.
33. $.08a=16$.
34. $2.5n=50$.
35. $200r=2.4$.
36. $1.4b=42$.
37. $.02y+.06y=36$.
38. $1000n=.001$.
39. $.001n=1000$.
40. $\frac{1}{5}b=7$.
41. $\frac{1}{2}x=97$.
42. $\frac{1}{6}k=6$.
43. $\frac{3}{5}n=21$.
44. $\frac{5}{6}n=30$.
45. $\frac{2}{7}x=5$.
46. $3.5m=70$.
47. $2.6m=23.4$.
48. $1.5t=60$.
49. $1\frac{1}{2}t=60$.
50. $1\frac{1}{4}x=100$.
51. $3\frac{2}{3}n=64$.
52. $\frac{1}{2}x+\frac{1}{3}x=20$.
53. $\frac{5}{12}x-\frac{1}{4}x=18$.
54. $1.06x+.08x=2508$.
55. $2\frac{3}{4}y-\frac{1}{4}y-20=0$.
56. $3\frac{1}{7}r-44=0$.
57. $5.65x+3.05x=21.75$.
58. $9n-22=16-3n$.
59. $48+24n=18n+288$.
60. $.01a+.001=.001a+.01$.

13. Translating sentences into equations. One of the principal reasons for studying algebra is to learn how to solve problems by the use of equations. One of the chief difficulties in solving problems is expressing the conditions of the problem in equations. This amounts to translating the English sentence into an equation. The pupils should have much practice in such translating.

Exercise 15

Express the following statements as equations.

1. If 4 is added to a number the result is 7.

SOLUTION. If n is the number, then $n+4=7$.

2. If 9 is subtracted from x the result is 20.

3. Seventeen times x is 35.

4. If 12 is subtracted from a the result is 46.

5. If x is divided by 6 the result is 17.

6. If 19 is subtracted from three times b the result is 40.

7. The sum of x and y is z.

8. If a is subtracted from b the remainder is r.

9. If m is subtracted from 4 times k the remainder is c.

10. There are s students in a school. If n leave there will be 34 remaining.

11. An investor has d dollars. He gains g dollars. He then has a dollars.

12. The perimeter of a square of side s is 18.

13. The perimeter of an equilateral triangle of side s is 24.

14. The perimeter of a rectangle of length l and width w is 200.

15. If 95 is subtracted from 42 times n the result is 658.

16. If x is added to its double the sum is 930.

17. One-fourth of n is 35.

18. Twenty hundredths of n is 8.

19. Twenty per cent of n is 8.

SUGGESTION. In such a case it is convenient to express the per cent as a decimal. The answer is then written, $.20n=8$.

20. Twelve per cent of x is 60.

21. A invests m dollars. He gains 20% of this investment. He then has \$480.

SOLUTION. $m+.20m=\$480.$

$1.20m=\$480.$

22. If 35% of n is added to n the sum is 540.

23. If 60% of x is subtracted from x the remainder is 428.

24. A man has d dollars and spends 40% of it. He then has \$90.

25. Sixty per cent of m is 4 more than b.

26. If 10% of x is added to 3 times x the sum is 372.

27. Twice n plus 15% of n plus 1% of n is 55.

28. George is x years old. In 6 years he will be 18 years old.

29. George's father is n years old. Ten years ago he was 28 years old.

30. William is b years old. In y years he will be 27 years old.

31. A man walks at the rate of 3 miles an hour, and travels m miles in 5 hours.

32. A man walks at the rate of 3 miles an hour and goes m miles in h hours.

33. A man walks at the rate of r miles an hour and goes m miles in h hours.

34. A train runs a distance of d miles in h hours when running at the rate of r miles an hour.

35. An automobile goes a distance of 139 miles in h hours while running at the rate of m miles an hour.

36. Write an equation that gives the distance, d, that an object moves in t seconds at the rate of r feet a second.

37. Write an equation that gives the number of seconds, t, in which an object will move a distance of d feet at the rate of r feet a second.

38. Write an equation that gives the number of feet, r, that a body travels in one second, if it travels d feet in t seconds.

39. When a is divided by b the quotient is q.

40. Twice n plus $\frac{2}{3}$ of n minus $\frac{1}{5}$ of n equals 146.

41. One-third of x plus 20% of x equals 98.

42. The cost of an article, c, plus 35% of the cost equals \$12.45.

43. One-half of a plus twice b is 5 less than c.

44. The quotient of 6 divided by n is 2 more than the product of r and s.

45. The sum of m and n is 4 more than $\frac{1}{3}$ of their product.

14. Solving problems by using equations. The ability to solve problems involving letters enables us to shorten the solutions of many problems, and also enables us to solve many problems that we could not solve before.

EXAMPLE. If 7 is added to three times a number the result is 52. Find the number.

SOLUTION. Let n = the number.
Then $3n$ = three times the number,
and $3n+7=52$.
Subtracting 7 from both members, $3n=45$.
Dividing both members by 3, $n=15$.

CHECK. $3\times15+7=52$.
$52=52$.

Exercise 16. Problems

1. What number increased by 22 equals 47?

2. If 5 is added to a number the result is 26. Find the number.

3. What number decreased by 24 equals 17?

4. If a certain number is multiplied by 8 the product is 216. Find the number.

5. One factor of 544 is 32 and the other is f. Find f.

6. A girl earns \$2.20 in $2\frac{3}{4}$ days. How much does she earn in one day?

7. A horse cost twice as much as a buggy. Together they cost \$540. Find the cost of each.

HINT. Let x represent the number of dollars that the buggy cost. Then what represents the number of dollars that the horse cost?

8. John has 4 times as much money as Charles. Together they have \$4.65. How much has each?

9. A man drives 196 miles in two days. The second day he drives $2\frac{1}{3}$ times as far as he did the first. How many miles did he drive each day?

10. The profit on a box of oranges is $\frac{1}{5}$ the cost. The selling price is \$4.80. What is the cost?

HINT. If c represents the number of dollars that the box of oranges costs, what represents the number of dollars profit? Then what represents the number of dollars in the selling price?

11. A number is increased by $\frac{1}{3}$ of itself. The result is 96. Find the number.

12. A regiment is increased by $\frac{1}{4}$ of its number and then contains 1280 men. How many did it contain at first?

13. A garden is 3 times as long as it is wide. If the width is x rods what is the length? What is the perimeter? If the perimeter is 40 rods, find the length and width.

14. One day a salesman sells d dollars' worth of goods, the next day twice as much, and the third day $\frac{2}{3}$ as much as the first day. The total sales for the three days were \$990. Find the amount of each day's sales.

15. A pair of boots is sold for \$7, thus gaining 40% of the cost. Find the cost.

HINT. If c is the number of dollars the boots cost, then $1.40c$ is the number of dollars the boots sell for.

16. An article is sold for \$13.30, which is 30% less than the cost. Find the cost.

17. Land is sold for \$154 an acre, thus making a profit of 10%. Find the cost.

18. One acute angle of a right triangle is 5 times as large as the other. How many degrees in each? Remember that the sum of the angles of a triangle equals two right angles.

19. One angle of a triangle contains n degrees, the second angle $\frac{1}{2}n$ degrees, and the third $\frac{3}{4}n$ degrees. How many degrees in each?

20. One of two complementary angles is $3\frac{1}{2}$ times as large as the other. How many degrees in each?

21. One angle of a triangle is twice as large as another. The third angle contains 30 degrees. How many degrees in each of the other angles?

22. A rope 40 ft. long is to be cut into two pieces so that one piece shall be $\frac{2}{3}$ as long as the other. How long will each piece be?

23. A motorcycle runs 5 laps around a track and then runs 12 miles to a certain town. Its cyclometer then shows that it has run $24\frac{1}{2}$ mi. How long is one lap around the track?

24. One side of a triangle is 1 in. longer than a second side, and the second side is 2 in. longer than the third side. The perimeter of the triangle is 15 inches. How long is each side?

25. A man invested a certain sum of money at 5%, one-third as much at 6%, and twice the first sum at 8%. His income from the three investments is \$69. How much did he invest at each rate?

26. The perimeter of an isosceles triangle is 18 inches. The base is two inches less than the sum of the two equal sides. How long is each side?

27. The area of a trapezoid is 60 sq. in. The altitude is 6 in. and the lower base is 4 in. longer than the upper base. Find the length of each base.

HINT. Use the formula for the area of a trapezoid.

28. The difference between the bases of a trapezoid is 6 in. and their sum is 34 in. The area of the trapezoid is 85 sq. in. Find the length of the altitude and of each base.

29. A man has a certain amount invested in $3\frac{1}{2}\%$ Liberty Bonds and twice as much in $4\frac{1}{4}\%$ Liberty Bonds. His income is \$1200. Find the amount invested in each kind of bonds.

15. Parentheses. The pupil has seen, page 32, First Book, that the order of operations in an expression is sometimes indicated by parentheses.

The expression $9+(8+6)$ means 9 plus the sum of 8 and 6, or 9 plus 14 ; $2(3+5)$ means 2 times the sum of 3 and 5, or 2 times 8 ; $4(7 \cdot 9)$ means 4 times the product of 7 and 9, or 4 times 63. Similarly, $a+(b+c)$ means a plus the sum of b and c ; $a(b+c)$ means a times the sum of b and c ; and $a(bc)$ means a times the product of b and c.

Exercise 17

State the meaning of each of the following :

1. $4+(5+6)$.
2. $4(5+6)$.
3. $4(5 \cdot 6)$.
4. $6(5+2+9)$.
5. $12(a+m+3)$.
6. $5(3m)$.
7. $x(y+z)$.
8. $m(xy)$.
9. $6+(r+s)$.
10. $12(a+m)+3$.
11. $12a+(m+3)$.
12. $(2+7)x$.

16. Definitions. Number expressions like 6, ab, or $7xy$ are called **terms** or **monomials.**

Number expressions which contain two or more terms are called **polynomials.** Thus, $a+b+c$, $m-4$, and $35+x+2y-z$ are polynomials.

A **binomial** is a polynomial of two terms ; as, $a+b$.

A **trinomial** is a polynomial of three terms; as, $2x-3n+h$.

17. Multiplication of monomials. The product $3 \cdot 5$ may be multiplied by 2 by multiplying either factor by 2. Thus, $2(3 \cdot 5)=6 \cdot 5=3 \cdot 10=30$. Similarly, $3(5 \cdot 4 \cdot 8)=15 \cdot 4 \cdot 8=5 \cdot 12 \cdot 8=5 \cdot 4 \cdot 24=480$.

Also the product of 3 and x, or $3x$, may be multiplied by 2 by multiplying either factor by 2. That is, $2(3x)=6x=3 \cdot 2x$. This product is usually written in the form $6x$.

These examples illustrate the

Rule. *To multiply a monomial by any number multiply any one of its factors, but only one, by the number.*

Exercise 18

Find the following products in at least two ways:

1. $2(3 \cdot 5)$.
2. $5(3 \cdot 7)$.
3. $4(3 \cdot 2)$.
4. $6(3 \cdot 4 \cdot 5)$.
5. $6(4 \cdot 3\frac{1}{3})$.
6. $10(5 \cdot 2\frac{1}{2})$.
7. $12(\frac{1}{3} \cdot \frac{1}{4})$.
8. $10(.3 \cdot 15)$.
9. $100(.02 \cdot 2.3)$.

Find the products of the following factors:

10. 2, $3x$.
11. 7, $4a$.
12. 3, $9m$.
13. 12, $40y$.
14. .5, $10x$.
15. 8, $2.1y$.
16. .3, $.8m$.
17. 6, $3.1416r$.
18. 2.3, $4.2r$.
19. 2, 3, x.
20. 2, 3, $5r$.
21. 5, 4, $2y$.
22. 4, 3 fives.
23. 8, 6 eights.
24. 8, $6 \cdot 8$.
25. 11, $5 \cdot 9$.

How many square inches in a rectangle:

26. $4''$ by $6''$?
27. a'' by b''?
28. $3''$ by x''?
29. $9''$ by $2x''$?
30. $2a''$ by b''?
31. $.5''$ by $3''$?

18. The order of factors. The order of factors in a product may be changed without changing the value of the product. Thus, $5 \times 8 = 8 \times 5$. $2 \times 7 \times 4 = 2 \times 4 \times 7 = 7 \times 4 \times 2$.

This principle is very useful in simplifying such products as $3m \cdot 4n$. By rearranging the factors and multiplying we get $3m \cdot 4n = 3 \cdot 4 \cdot m \cdot n = 12mn$.

Exercise 19

Find the products of the following factors:

1. 3, $5x$.
2. 2, 3, $7t$.
3. $3x$, $4z$.
4. $5m$, $20n$.
5. $3x$, y.
6. $10a$, $4b$.
7. a, $7b$.
8. x, $20y$.
9. $2r$, $3s$.
10. $4t$, $5m$.
11. $\frac{1}{2}m$, $4n$.
12. $\frac{1}{3}x$, $\frac{2}{3}y$.
13. 4, $2a$, b.
14. 6, $3a$, $5b$.
15. $.03s$, $.2t$.
16. .1, $.06k$.

How many cubic inches in a rectangular solid:

17. 3 in. by 4 in. by 10 in.?
18. 5 in. by 5 in. by x in.?
19. a in. by b in. by c in.?
20. $2a$ in. by $3b$ in. by c in.?
21. $3x$ in. by 26 in. by z in.?
22. .2 in. by 30 in. by m in.?
23. $25a$ in. by $4b$ in. by c in.?
24. $\frac{3}{4}x$ in. by $\frac{1}{8}y$ in. by $\frac{1}{2}z$ in.?

19. Exponents and powers. You have learned that $4 \cdot 4$ may be written 4^2 and that $a \cdot a$ may be written a^2. Similarly 2^3 means $2 \cdot 2 \cdot 2$ or 8, and a^3 means $a \cdot a \cdot a$. We may indicate that 3 is used as a factor 5 times in either of the two ways $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ or 3^5. Which is the shorter way? Find the product.

A **power** of a number is the product obtained by using the number as a factor one or more times. Thus, 3^4 or $3 \cdot 3 \cdot 3 \cdot 3$ or 81 is a power of the number 3.

We read a^2 as "a square"; a^3 as "a cube"; and a^4 as "a exponent 4" or "a fourth." In a^4, 4 is called the **exponent** and a the **base** of the power.

When the exponent is 1 it is usually omitted. Thus a^1 is written a.

Since a^2 means $a \cdot a$ and a^3 means $a \cdot a \cdot a$, then $a^2 \cdot a^3 = a \cdot a \cdot a \cdot a \cdot a = a^5$. Similarly $3^5 \cdot 3^2 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^7$.

The pupil should observe the difference between $2x^3$ and $(2x)^3$. $2x^3$ means $2 \cdot x \cdot x \cdot x$, and $(2x)^3$ means $2x \cdot 2x \cdot 2x = 8x^3$.

Rule. *The product of powers having the same base equals that base with an exponent equal to the sum of the exponents of the powers.*

EXAMPLE 1. Find the value of 4^3.

SOLUTION. $4^3 = 4 \cdot 4 \cdot 4 = 64$.

EXAMPLE 2. Multiply 2^4 by 2^3.

SOLUTION. $2^4 \cdot 2^3 = 2^7 = 128$.

EXAMPLE 3. Multiply a^2b by $a^3c^2b^2$.

SOLUTION. $a^2b \cdot a^3c^2b^2 = a^2 \cdot a^3 \cdot b \cdot b^2 \cdot c^2 = a^5b^3c^2$.

Exercise 20

Write the following, using exponents:

1. $3 \cdot 3 \cdot 3$.
2. $5 \cdot 5 \cdot 5 \cdot 5$.
3. $b \cdot b \cdot b$.
4. $7 \cdot a \cdot a \cdot a$.
5. $m \cdot m \cdot r \cdot r \cdot r$.
6. $5 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$.

7. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$.
8. $2 \cdot a \cdot a \cdot n \cdot n \cdot n$.
9. $a \cdot n \cdot c \cdot c$.
10. $x \cdot x \cdot y \cdot z \cdot z \cdot z$.
11. $x \cdot 4 \cdot 4 \cdot z \cdot z \cdot z$.
12. $7 \cdot 10 \cdot 10 \cdot 10$.
13. $3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$.
14. $10 \cdot 10 \cdot 10 \cdot a \cdot a \cdot a$.

Find the value of the following :

15. 2^4. 16. 3^3. 17. 5^2. 18. 7^3. 19. 4^5. 20. 2^7. 21. 10^3.
22. 10^4. 23. 10^5. 24. 10^8. 25. 2^{10}. 26. 5^5. 27. 25^3.

Write these powers of 10, using exponents :

28. 100. 29. 10,000. 30. 1,000,000.
31. 1,000. 32. 1,000,000,000. 33. 100,000.

Write the following without exponents, then multiply :

34. 10^4. 35. 5^3. 36. 12^2. 37. $.1^4$. 38. $(.06)^2$.
39. $(.01)^4$. 40. $(\frac{2}{3})^2$. 41. $(\frac{3}{4})^3$. 42. 2.5^5. 43. $(1\frac{1}{2})^3$.

Multiply the following :

44. $2^3 \cdot 2^4$. 45. $3^2 \cdot 3^5$. 46. $10^4 \cdot 10^2$.
47. $.1^5 \cdot .1^3$. 48. $a^3 \cdot a^5$. 49. $3 \cdot 3^4$.
50. $a^2 \cdot a$. 51. $c \cdot c^3$. 52. $a^2 \cdot b^2 \cdot a^3 \cdot b^5$.
53. $m^2n \cdot mn^2$. 54. $2c^3n \cdot 5c$. 55. $7ab^2c \cdot 7^2c^2$.

20. Multiplying a polynomial by a monomial.

1. Using dots, show the sum of 5 dots and 7 dots.
2. With dots show 3 times the sum of 5 dots and 7 dots.
3. What is the meaning of $3(5+7)$?
4. In this group each row contains $(5+7)$ dots.

The 3 rows contain $3(5+7)$ dots. Show ····· ····· that the left-hand group contains $3 \cdot 5$ dots ····· ····· and the right-hand group contains $3 \cdot 7$ dots. ····· ·····

This shows that $3(5+7) = 3 \cdot 5 + 3 \cdot 7$.

5. Show with dots that $3(2+4+5) = 3 \cdot 2 + 3 \cdot 4 + 3 \cdot 5$.

These examples illustrate the following

Rule. *To multiply a polynomial by a monomial multiply each term of the polynomial by the monomial and add the products.*

Exercise 21

Find the following products :

1. $4(6+7)$.
2. $3(a+6)$.
3. $7(x+y)$.
4. $5(2x+3y)$.
5. $a(x+y)$.
6. $2a(a+n)$.
7. $2\pi(r+R)$.
8. $7(3m+4)$.
9. $16(3a+2b)$.
10. $a(b+c+d)$.
11. $3x(m+n+3)$.
12. $5r(2x+4y+10)$.
13. $a(a+b)$.
14. $2a(3c+5a)$.
15. $r^2(r+3)$.
16. $3n^2(2m+n)$.
17. $c^3(c+g)$.
18. $\frac{1}{2}h(b+b')$.

21. Multiplication by a polynomial. In multiplying 46 by 32 we get the products $30 \cdot 6$, $30 \cdot 40$, $2 \cdot 6$, and $2 \cdot 40$ and add them. Let the pupil show that the following are correct :

1. $(2+3)(6+9) = 2 \cdot 6+2 \cdot 9+3 \cdot 6+3 \cdot 9$.
2. $65 \cdot 87 = (60+5)(80+7) = 60 \cdot 80+60 \cdot 7+5 \cdot 80+5 \cdot 7$.
3. $48 \cdot 391 = (40+8)(300+90+1) =$
$40 \cdot 300+40 \cdot 90+40 \cdot 1+8 \cdot 300+8 \cdot 90+8 \cdot 1$.
4. $(m+n)(x+y) = mx+my+nx+ny$ when $m=3$, $n=8$, $x=4$, $y=12$.

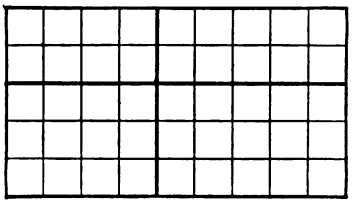


FIG. 7

Such products can be illustrated by rectangles.

Figure 7 shows that a rectangle $2+3$ units wide and $4+5$ units long contains $2 \cdot 4+2 \cdot 5+3 \cdot 4+3 \cdot 5$ square units.

The above examples illustrate the following

Rule. *To find the product of two polynomials multiply each term of one by each term of the other and add the products.*

EXAMPLE. Multiply $3m+5n$ by $2m+n$.

SOLUTION.

$$\begin{array}{r} 3m+5n \\ 2m+\ n \\ \hline 6m^2+10mn \\ \quad 3mn+5n^2 \\ \hline 6m^2+13mn+5n^2 \end{array}$$

Exercise 22

Find the following products :

1. $(10+8)(20+4)$.
2. $(a+x)(b+y)$.
3. $(x+2)(x+3)$.
4. $(a+7)(a+9)$.
5. $(r+1)(r+5)$.
6. $(s+3)(s+7)$.
7. $(2m+3)(3m+5)$.
8. $(a+6)(a+9)$.
9. $(3x+8)(4x+1)$.
10. $(a+b)(a+b)$.
11. $(r+s)(r+s)$.
12. $(n+2)(n+2)$.
13. $(2x+1)(2x+1)$.
14. $(2m+n)(m+2n)$.
15. $(6m+7a)(2m+3a)$.
16. $(a^2+2)(a^2+3)$.
17. $(n^2+5)(n^2+7)$.
18. $(2r^2+1)(r^2+3)$.
19. $(x+2)(4x+7)$.
20. $(a+3)^2$.

22. Definition of division. **Division** is the process of finding one of two factors when their product and the other factor are given.

Thus, to divide 18 by 3 means to find the factor by which 3 must be multiplied to get 18. Then $18 \div 3 = 6$, since $6 \cdot 3 = 18$. In the same way $12mn \div 3m = 4n$, since $3m \cdot 4n = 12mn$.

Exercise 23

Find what the first number must be multiplied by to get the second :

1. 4, 20.
2. 48, 6.
3. $2x$, $6x$.
4. $2m$, $12m$.
5. 2, $6x$.
6. a, $3a$.
7. a, a^2.
8. a^2, a^2.
9. a^2, $4a^2$.
10. a^2, a^3.

Divide the first number by the second in the following :

11. 18, 6.
12. $3 \cdot 18$, 6.
13. $18 \cdot 24$, 6.
14. $10 \cdot 7$, 5.
15. ab, a.
16. $3ab$, b.
17. $5mn$, mn.
18. $12mn$, m.
19. $12mn$, $3m$.
20. $15rs \div 5s$.
21. $a^2 \div a$.
22. $3b^2 \div b$.
23. $b^3 \div b$.
24. $b^3 \div b^2$.
25. $6 \div \frac{1}{2}$.
26. $6a \div \frac{1}{2}$.
27. $6a \div \frac{1}{2}a$.
28. $\frac{3}{4}r^2 \div \frac{1}{4}r$.
29. $\frac{2}{3}ab \div 3$.
30. $\pi r^2 \div r^2$.
31. $2\pi r \div 2\pi$.
32. $.1n \div .01$.
33. $.01n \div .1n$.
34. $25m^2n \div 5mn$.

23. Division of a polynomial by a monomial. In what two ways may $6 \cdot 4$ be multiplied by 2 without first multiplying 6 by 4?

Then in what two ways may $6 \cdot 8$ be divided by 2 without first multiplying 6 by 8?

How may $7+5$ be multiplied by 3 without first adding 7 and 5?

Then how may $21+15$ be divided by 3 without first adding 21 and 15?

Summary. *To multiply, or to divide, a monomial by any number, multiply, or divide, any* ONE *of its factors by the number.*

To multiply, or to divide, a polynomial by any number, multiply, or divide, EACH *term of the polynomial by the number.*

EXAMPLE. Divide $12x^3y+8x^2y^2+6xy^3$ by $2xy$.

SOLUTION.
$$2xy)\underline{12x^3y+8x^2y^2+6xy^3}$$
$$6x^2+4xy+3y^2$$

Exercise 24

1. Multiply $6 \cdot 9$ by 3 in three different ways.
2. Divide $18 \cdot 36$ by 6 in three different ways.
3. Divide $6 \cdot 14 \cdot 8$ by 7 in the easiest way.
4. Multiply $6+8$ by 5 in two ways.
5. Divide $18+36$ by 6 in two ways.

6. $(9 \cdot 7+6 \cdot 7) \div 7$.
7. $(ac+bc) \div c$.
8. $(6a+6b) \div 6$.
9. $(12m+6n) \div 3$.
10. $(30x+6) \div 6$.
11. $(bx+cx) \div x$.
12. $(ax+an) \div a$.
13. $(a^2+2a) \div a$.
14. $(5c^2+7c) \div c$.
15. $(5a+10b+45c) \div 5$.
16. $(400+60+8) \div 2$.
17. $(ab+ac+ad) \div a$.
18. $(30a^2+66a+48) \div 6$.
19. $(20x^3+32x^2+8x) \div 4x$.
20. $(5 \cdot 10^3+2 \cdot 10^2+3 \cdot 10) \div 10$.
21. $(3x^2+x) \div x$.
22. $(10a+5) \div 5$.
23. $(x^3+x^2+x) \div x$.
24. $(2ab+b^2) \div b$.
25. $(.2m^2+.6m) \div .1m$.
26. $(\pi r^2+2\pi r) \div r$.

24. Solving formulas. The pupil has seen that the formula is a very helpful means for finding the value of the letter in its left member when the values of the letters in its right member are given. Thus, if the two bases and the altitude of a trapezoid are known to be 12 ft., 8 ft., and 3 ft., the area of the trapezoid may be found by substituting these values in the formula

$$T=\tfrac{1}{2}h(b+b').$$

But a problem may be given in which the area, the altitude, and the upper base are known and the length of the lower base desired. We then need a formula in which the upper base, b, stands alone in the left member. We may get such a formula by using the formula $T=\frac{1}{2}h(b+b')$ as an equation in which b is the unknown number and the other letters treated as known numbers. It will be seen from the examples given below that any letter in a formula may be regarded as the unknown, and its value may be found in terms of the other letters by solving the equation for the unknown letter.

EXAMPLE 1. From the formula $s=c+g$ get a formula for the cost, c, when the selling price, s, and the gain, g, are known.

SOLUTION. $s=c+g.$

Subtract g from each member, then $s-g=c,$

or $c=s-g.$

EXAMPLE 2. Solve the formula $T=\frac{1}{2}h(b+b')$ for b.

SOLUTION. $T=\frac{1}{2}h(b+b').$

Multiply each member by 2. $2T=h(b+b').$

Performing the multiplication indicated in the right member,

$2T=hb+hb'.$

Subtract hb' from each member. $2T-hb'=hb.$

Divide each member by h. $\dfrac{2T-hb'}{h}=b,$

or $b=\dfrac{2T-hb'}{h}.$

Exercise 25

1. From the formula $s=c+g$ get a formula for finding the gain when the selling price and the cost are known.

2. Write the formula for the area of a rectangle when the base and the altitude are known. From it get a formula for finding the base when the area and the altitude are known.

3. Solve the formula $T=\frac{1}{2}ab$ for a; for b.

SUGGESTION. First multiply each member by 2.

4. Use one of the formulas found in the preceding exercise to find the altitude of a triangle whose area is 36 sq. in. and whose base is 4 in.

5. Find the base of a triangle whose area is 141 sq. rd. and altitude 77 yd.

6. Write the formula for finding the interest on a given sum of money for a given time at a given rate. Solve this formula for the principal, p, supposing the interest, i, the time, t, and the rate, r, to be known.

7. State the rule given by the formula $p=\frac{i}{rt}$. Find the principal which will produce \$400 interest in 2 yr. 6 mo. at 5%.

8. Solve the formula $i=prt$ for r. Find the rate of interest at which \$456 will produce \$95.76 interest in 3 yr.

9. Solve the formula $i=prt$ for t. Find the time required for \$1275 to produce \$56 interest at 6%.

10. State as a rule the formula for finding the time in which a given principal will produce a given interest at a given rate.

11. A man bought a bond for \$110 and after two years sold it for \$115. During that time his income from the bond was \$4 a year. He also bought a farm for \$1500 which he kept for $1\frac{1}{2}$ years and sold for \$1550. His net income from the farm during the time that he held it was \$114. On which

investment did he receive the higher rate of interest on the money invested?

12. Write the formula for finding the dividend, D, when given the divisor, d, the quotient, q, and the remainder, r. Solve this formula for r; for q; for d.

13. Make a formula for finding the perimeter, p, of a rectangle whose length is l and width w. Solve the formula for l; for w.

14. Solve the formula $a=2x+x$ for x.

15. Solve the formula $s=b+rb$ for b.

HINT. First add the terms containing b. $1\,b+rb=(1+r)b$.

16. Solve the formula $a=p+prt$ for p.

17. Use the new formula you found in the preceding problem to find the principal which will amount to \$660 if put at interest for 2 yr. at 5%.

18. Solve for r the formula $a=p+prt$. If \$2000 is put at interest for 4 yr. 6 mo. at a certain rate it amounts to \$2360. Find the rate.

19. Solve the formula $F=\frac{9}{5}C+32$ for C.

20. Use the formula given in the preceding exercise and find F when $C=0$; when $C=16$.

21. Use the result you obtained in exercise 19 and find C when $F=98$; when $F=32$.

22. The formula for the average, a, of the two numbers r and s is $a=\frac{r+s}{2}$. Solve this equation for s.

23. The average of two numbers is 85. One of the numbers is 76. What is the other?

24. The average of two numbers is $\frac{3}{4}$. One of the numbers is $\frac{2}{3}$. What is the other?

25. State in words the rule for finding the average of two numbers.

Exercise 26. Review

Write the following products, using exponents :

1. $a \cdot b \cdot a \cdot c \cdot a \cdot c$.
2. $6 \cdot m \cdot n \cdot 6 \cdot m \cdot n$.
3. $\pi r \cdot \pi r$.
4. $4 \cdot a \cdot b \cdot b \cdot 4 \cdot a$.
5. $x \cdot y \cdot x \cdot x \cdot y \cdot y$.
6. $r \cdot s \cdot s \cdot t \cdot r \cdot t \cdot s$.

Write the following without exponents :

7. $6a^2b^3$.
8. m^4n^3.
9. 10^4.
10. $7^2r^6s^3t$.
11. $\dfrac{2^3 \cdot 5^3}{10^3}$.
12. $(\frac{2}{3})^3 \cdot (\frac{5}{6})^2$.

Simplify :

13. $2x+6+8x+10$.
14. $12x+9y-2x-6y$.
15. $3(2x+4)-9$.
16. $\pi r(\pi r^2+6)$.
17. $(m+n)(m+n)$.
18. $(10a+b)(10\,a+b)$.
19. $3x(2x^3+6x^2+x+8)$.
20. $(20+5)(40+7)$.
21. $(a^2+2ab+b^2)(a+b)$.

Divide :

22. $900+60+3$ by 3 in two ways.
23. $2mn+n^2$ by n.
24. rs by xy.
25. 365 by 10^3.
26. $2a^3+13a^2+a$ by a.
27. $\frac{3}{4}x+\frac{2}{3}y+2$ by $\frac{1}{2}$.
28. m by n.
29. 6 by a.
30. $4\pi r$ by πr^2.

31. In a room there are m men, w women, and 17 children. How many persons in all? If 8 persons leave, how many remain? If a more persons leave, how many remain?

32. A boy has c cents. How many more cents must he get to have one dollar?

33. A boy has x cents. How many more must he get to have n dollars?

34. How many hours are required to walk $12x^2$ miles at the rate of $3x$ miles per hour?

35. How many hours are required to walk x miles at the rate of $5m$ miles per hour?

36. What must be added to $4y-9$ to give $4y$? To $a-b$ to give a?

37. What does it mean to say that a number satisfies an equation?

38. Define root of an equation.

39. What does it mean to solve an equation?

40. What does it mean to check a solution?

41. State four principles used in solving equations. Give equations in the solution of which these principles are used.

42. Define division.

Solve the following equations :

43. $2x-9=12$.

44. $15n=42+6n$.

45. $.5y+\frac{1}{20}=.02y+1.01$.

46. $y+.25y=17$.

47. $3x+\dfrac{5}{2\frac{1}{2}}=x+2.5$.

48. Solve for x, $ax=b$.

49. Solve for x, $ax+bx=c$.

50. Solve for a, $ma-na=p$.

51. Solve for r, $ar=12+r$.

52. Solve for y, $y-10=mr$.

53. This formula is used in finding the load that may safely be placed upon wooden pillars :

$$L=\frac{1125}{1+\dfrac{l^2}{1100d^2}}$$

In this formula L represents tons and l and d represent inches. Find L if $l=96$ and $d=8$; also if $l=120$ and $d=8$.

54. Show that the area of Figure 8 is given by the formula $A=dt+(s+y)z$. Find A if $d=12$ in., $t=2$ in., $z=4$ in., $y=1\frac{1}{2}$ in., and $s=\frac{3}{4}$ in.

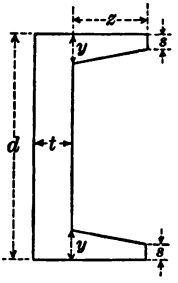


FIG. 8

55. $I=0.0491(a^4-b^4)$. Find I when $a=6$ and $b=5$; also when $a=1.5$, and $b=1.2$, correct to 0.0001.

56. If $y=5x^2-4x+6$, find the value of y for each of the following values of x : 0, 1, 5, 10, 20.

57. Show that the area of the picture frame represented in Figure 9 is $A=ld+wd-4d^2=d(l+w-4d)$. Find A if $l=18$ in., $w=16$ in., and $d=2\frac{1}{2}$ in.; also if $l=12$ in., $w=8\frac{1}{2}$ in., and $d=1\frac{1}{4}$ in.

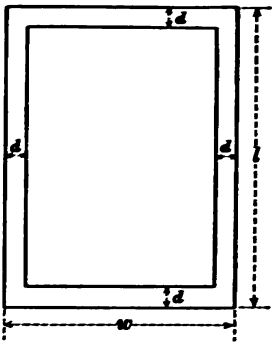


FIG. 9

58. $m=\dfrac{l+r}{l-r}$. Find m when $l=10$, $r=6$; when $l=4.8$, and $r=3.2$; when $l=375$, and $r=.025$.

59. Solve $c=2\pi r$ for r. Find r correct to .01, if $c=120$ ft. Use $\pi=3\frac{1}{7}$.

60. Make a formula which gives the perimeter, p, of an isosceles triangle one of whose equal sides is l and whose base is b. Solve this formula for l. Find l if $p=90$ ft. and $b=18$ ft.

61. Solve for c, $c+.25c=875$.

62. Solve for c, $c-15\%$ of $c=3.91$.

63. Solve for s, $s-1.2\%$ of $s=829.92$.

64. By the breaking strength of a rope is meant the least pull that is likely to break it. The breaking strength of manila rope is 3000 lb. per square inch of cross section area. Make a formula stating this fact, where I is the breaking strength and d the diameter of the rope.

65. What weight would be likely to break a manila rope 1 in. in diameter? One $1\frac{1}{2}$ in. in diameter?

CHAPTER III

SQUARE ROOT AND ITS APPLICATIONS

25. To find the length of one side of a square whose area is known. The area of a square is 9 sq. in. How are these 9 square inches arranged?

Since the number in a row is the same as the number of rows, we must find the number whose square is 9. This is 3. Therefore there are 3 rows of square inches, each row containing 3 square inches. Therefore each side of the square is 3 inches long.

The number whose square is 9 is called the **square root** of 9. It is expressed thus : $\sqrt{9}$. The symbol $\sqrt{\ }$ is called a **radical sign.**

The square root of a number is one of the two equal factors of the number.

Thus, 5 is the square root of 25; 8 is the square root of 64; and 10 is the square root of 100.

EXAMPLE. Find the length of one side of a square whose area is 49 square inches.

SOLUTION. The area is equal to 49 sq. in.
Therefore the length of one side $= \sqrt{49}$ inches.
$\sqrt{49} = 7$.
Therefore the length of one side $= 7$ inches.

In finding the square roots of numbers it is useful to know the squares of the integers from 1 to 25 inclusive. The pupils should commit them to memory and be able to repeat them in 1 minute.

Exercise 27

1. $\sqrt{16}=?$	9. $\sqrt{121}=?$	17. $\sqrt{361}=?$
2. $\sqrt{81}=?$	10. $\sqrt{64}=?$	18. $\sqrt{400}=?$
3. $\sqrt{4}=?$	11. $\sqrt{25}=?$	19. $\sqrt{196}=?$
4. $\sqrt{36}=?$	12. $\sqrt{169}=?$	20. $\sqrt{576}=?$
5. $\sqrt{144}=?$	13. $\sqrt{324}=?$	21. $\sqrt{484}=?$
6. $\sqrt{100}=?$	14. $\sqrt{225}=?$	22. $\sqrt{529}=?$
7. $\sqrt{49}=?$	15. $\sqrt{9}=?$	23. $\sqrt{441}=?$
8. $\sqrt{256}=?$	16. $\sqrt{289}=?$	24. $\sqrt{625}=?$

Find the length of the side of the square whose area is:

25. 81 sq. rd. 27. 49 sq. yd. 29. 324 sq. rd.
26. 169 sq. in. 28. 144 sq. ft. 30. 625 sq. in.
31. 289 two-inch squares. 32. 121 one-half-inch squares.
33. $\frac{4}{9}$ of a sq. ft. 34. 25 sq. rd. 35. 10 acres.

26. To find the square root of larger numbers by factoring. If the square root of 36 were forgotten it could be found thus:

$36=2\cdot 18=2\cdot 3\cdot 6=2\cdot 3\cdot 3\cdot 2=(2\cdot 3)(2\cdot 3)=6^2$.

Therefore $\sqrt{36}=6$.

Rule. *Break the number up into factors, then arrange these factors, if possible, into two groups having the same factors. The product of the factors of one of these groups is the square root of the number.*

EXAMPLE. $\sqrt{1764}=?$

SOLUTION. $1764=2\cdot 2\cdot 3\cdot 7\cdot 3\cdot 7$
$=(2\cdot 3\cdot 7)(2\cdot 3\cdot 7)=42^2$.
Therefore $\sqrt{1764}=42$.

2)1764
2)882
3)441
3)147
7)49
7

The solution may be shortened if the pupil sees that 441 equals $21\cdot 21$. He may then write

$1764=2\cdot 2\cdot 21\cdot 21=(2\cdot 21)(2\cdot 21)=42^2$.

Exercise 28

Find by factoring the square roots of the following :

1. 324.	7. 900.	13. 2704.	19. 40,000.
2. 225.	8. 2025.	14. 2304.	20. 250,000.
3. 729.	9. 1225.	15. 3969.	21. 19,600.
4. 441.	10. 2401.	16. 3249.	22. 67,600.
5. 676.	11. 1024.	17. 7056.	23. 30,976.
6. 1089.	12. 3136.	18. 8281.	24. 99,225.

27. The square of the sum of two numbers.

1. Write the sum of a and b.
2. Indicate the square of the sum of a and b.
3. By multiplying find the square of $a+b$.

EXAMPLE 1. $(x+y)^2=?$

$$(x+y)^2=(x+y)(x+y)$$
$$=x^2+2xy+y^2$$

$$\begin{array}{r} x+y \\ x+y \\ \hline x^2+xy \\ \quad xy+y^2 \\ \hline x^2+2xy+y^2 \end{array}$$

4. $(m+n)^2=?$ Find by multiplication.
5. $(s+t)^2=?$ Write the result without multiplication.

These illustrate the principle: *The square of the sum of two numbers equals the square of the first number plus two times the product of the two numbers plus the square of the second number.*

Stating this principle as a formula,

$$(a+b)^2=a^2+2ab+b^2.$$

EXAMPLE 2. Find the square of the sum of 9 and 6.

SOLUTION. $(9+6)^2=9^2+2\cdot 9\cdot 6+6^2$
$=81+108+36$
$=225.$

EXAMPLE 3. Find the square of 78.

SOLUTION. $78^2=(70+8)^2=70^2+2\cdot 70\cdot 8+8^2$
$=6084.$

CHECK. $78\times 78=6084$ by actual multiplication.

Exercise 29

1. Indicate the sum of n and s; the square of the sum of n and s.

2. $(a+2)^2$ indicates the square of the sum of what two numbers?

3. $(4\frac{2}{3})^2$ indicates the square of the sum of what two numbers?

Using the principle above find the squares of the following sums:

4. $(t+u)^2$.
5. $(4+k)^2$.
6. $(n+2)^2$.
7. $(5+7)^2$.
8. $(30+4)^2$.
9. $(5\frac{1}{2})^2=(5+\frac{1}{2})^2=?$
10. $(2\frac{1}{2})^2$.
11. $45^2=(40+5)^2=?$
12. 32^2.
13. 74^2.
14. 63^2.
15. $(6\frac{1}{4})^2$.
16. $(3\frac{1}{3})^2$.

17. The expression $a^2+2ab+b^2$ is the square of the sum of what two numbers? How is the first number found? How can the second number be found? Answer the same questions for $x^2+2xy+y^2$.

18. $m^2+2mn+n^2=(?+?)^2$. Check by squaring the result.

19. $4^2+2\cdot 5\cdot 4+5^2=(?+?)^2$.

20. $4x^2+20xy+25y^2=(?+?)^2$.

21. In the expression $a^2+2ab+b^2$, $2ab$ is twice the product of the two numbers whose sum was squared. What can be done to $2ab$ to find the product of the two numbers? If you knew one of the numbers how could you then find the other?

22. In the expression $3^2+30+(?)^2$, 30 is twice the product of the two numbers whose sum was squared. What is their product? What is the first number? How can you find the second? Supply the missing third term of the expression.

Supply the missing numbers in these equations:

23. $3^2+2\cdot 3\cdot 4+(?)^2=(?+?)^2$.

24. $25+10\cdot 3+(?)^2=(?+?)^2$.

25. $36+60+(?)^2=(?+?)^2$.

Rule for finding the two numbers when given the first two terms of the square of their sum. *Find the first number by taking the square root of the first term given. To find the second number divide the second term given by twice the first number, which has just been found.*

26. $400+40+(?)^2=(?+?)^2$.

27. $100+160+(?)^2=(?+?)^2$.

28. $900+240+(?)^2=(?+?)^2$.

29. $3600+840+(?)^2=(?+?)^2$.

30. $520+(?)^2=400+120+(?)^2=(?+?)^2$.

31. $1140+(?)^2=(?+?)^2$.

HINT. The largest number of hundreds in 1140 which is a square number is 900. Separate 1140 into the two terms 900+240.

In the following, first separate the given number into two terms so that the first shall be the largest number of hundreds which is a square.

32. $560+(?)^2=(?+?)^2$.

33. $2800+(?)^2=(?+?)^2$.

34. $280+(?)^2=(?+?)^2$.

35. $1200+(?)^2=(?+?)^2$.

36. $680+(?)^2=(?+?)^2$.

37. $3200+(?)^2=(?+?)^2$.

38. $1840+(?)^2=(?+?)^2$.

In finding the square root of any number we separate it into three terms as in the expression $a^2+2ab+b^2$, whose square root we know how to find. Sometimes the first term is given separate but the last two terms are given combined. Thus, $400+129=(?+?)^2$. In this case we do not know how much of 129 is $2ab$ and how much is b^2. However, if a is a large number compared with b, then $2ab$ is much larger than b^2, and we can find b approximately by dividing all of 129 by $2a$, that is by $2 \cdot 20$. Dividing 129 by 40 gives 3 approximately. Then $2ab+b^2=2 \cdot 20 \cdot 3+3^2=129$. And $400+129=(20+3)^2=400+120+9$.

Supply the missing numbers in the following :

39. $4900+284=(?+?)^2$.
40. $1600+81=(?+?)^2$.
41. $6400+489=(?+?)^2$.
42. $2500+416=(?+?)^2$.
43. $900+396=(?+?)^2$.
44. $400+384=(?+?)^2$.
45. $1600+704=(?+?)^2$.
46. $3600+1161=(?+?)^2$.

28. The number of orders in the square of a number. To find the square root of a number like 1849 it is necessary to learn how to separate it into terms so that the plan of the preceding exercise may be followed. When squaring a number such as 43 we really separate it into 40+3, that is, into its tens and units. We must now study the number of orders in the squares of the different numbers.

Fill the blanks in the following :

$1^2=$. $10^2=$. $100^2=$.
$9^2=$. $99^2=$. $999^2=$.

How many orders in the numbers 1 and 9? In 10 and 99? In 100 and 999?

How many orders in the squares of numbers of one order? How many orders in the squares of numbers of two orders? In the squares of numbers of three orders?

These examples illustrate the

Principle. *The number of orders in the square of a number is twice as many as in the number, or one less than twice as many.*

It follows from this rule that if a square number contains an even number of orders, its square root contains one-half as many orders. If a square number contains an odd number of orders add 1 to the number of orders and divide the sum by 2, to get the number of orders in the square root. Thus the square root of 16,777,216 contains 4 orders, and the square root of 27,225 contains 3 orders.

Exercise 30

1. Tell how many orders there are in the square of each of the following numbers : 8, 3, 40, 98, 200, 7, 46, 345.

2. Give the square root of each of the following : 9, 49, 64, 81.

3. If a number contains one or two orders, how many orders are there in its square root?

4. Give the square root of each of the following : 400, 225, 900, 625, 100, 6400, 8100, 4225. How many orders in each of these numbers? How many orders in the square root of each?

5. Without finding the square roots of the following numbers tell how many orders there are in each square root: 289, 841, 6084, 4, 36, 7921, 27889, 35344, 148996, 606841.

29. Finding the squares of units, tens, and hundreds. Let the pupil fill the blanks in the following :

$3^2 =$.	$30^2 =$.	$300^2 =$.
$8^2 =$.	$80^2 =$.	$800^2 =$.

These examples illustrate the fact that the square of units' digit is found in units' and tens' orders, the square of tens' digit is found in hundreds' and thousands' orders, and that the square of hundreds' digit is found in ten-thousands' and hundred thousands' orders.

In finding the square roots of numbers we shall first want to know how many orders there are in the square root. Thus, if we want to find the square root of 86436, we first see that the square root contains three orders. Why? What are the names of these orders? We know that the square of the number of hundreds in this root is found in ten-thousands. The largest square in 8 is 4. The number of hundreds in the square root is then 2. The places of the squares of the orders of the root are usually indicated by pointing off the number whose square root is to be found into periods of two figures each, beginning at the right, thus, 8'64'36.

30. To find the square root of any integer. We wish to find the square root of 1849. We point off 1849 into periods of two figures each and find that there will be two orders in the root.

The square of the tens must be found in the 1800, and the largest number of tens whose square is found in 1800 is 4 tens, or 40. Its square is 1600. We then separate 1849 into 1600+249 and finish the work as in example 39 of exercise 29.

Exercise 31

Find the square root of each of the following :

1. 1089.	2. 3844.	3. 5625.	4. 2809.	5. 5329.
6. 784.	7. 3136.	8. 8464.	9. 1521.	10. 256.

The pupil will now find convenient the following form for computing square roots.

11. Find the square root of 3969.

$$\begin{array}{r|l} & a+b \\ 3969 & 60+3 \\ a^2=3600 & \\ \hline 2a=120 & 369 \\ b=\ 3 & \\ \hline 2a+b=123 & 369 \end{array}$$

SOLUTION. The trial divisor, Find b by dividing 369 by 120. Since $(2a+b)b=2ab+b^2$, 3 is added to 120 and the sum multiplied by 3.

In practice this form is much condensed.

$$\begin{array}{r|l} 39'69 & 63 \\ 36 & \\ \hline 123 & 369 \\ & 369 \end{array}$$

Find the greatest square in 39 to be 36.

Place the square root of 36 as the first figure in the root.

Subtract the 36 and bring down the next *two* figures.

For trial divisor use $2 \cdot 6$. Divide the remainder, omitting the final digit 9, by the trial divisor. $36 \div 12 = 3$.

Annex this figure to the root, also to the trial divisor.

Multiply this complete divisor by the new figure of the root.

12. Find the square root of 57121.

SOLUTION. Separate the number into periods.

Find the greatest square in 5 and subtract it.

Bring down the next period.

Divide 17 by the trial divisor 2 · 2.

Annex the quotient 4 to the trial divisor, also to the figure of the root already found.

Multiply the complete divisor by the new figure of the root.

The product is more than 171, proving that 4 is too large. Try 3.

For the next trial divisor double the part of the root already found, 23, and divide 422 by the product.

Annex the quotient 9 to the trial divisor and also to the root.

Multiply the complete divisor by the new figure of the root and subtract as before.

```
5'71'21)239
    4
43 | 171
     129
469 | 4221
      4221
```

Find the square root of each of the following :

13. 2209.	**16.** 46225.	**19.** 94249.	**22.** 516961.
14. 54756.	**17.** 795664.	**20.** 39204.	**23.** 401956.
15. 58081.	**18.** 182329.	**21.** 804609.	**24.** 11641744.

31. To find the square root of a decimal fraction.

Fill the blanks in the following :

$.1^2 =$.	$.01^2 =$.	$.001^2 =$.
$.9^2 =$.	$.09^2 =$.	$.009^2 =$.

The square of a number having one decimal order has how many decimal orders? Of a number having two decimal orders? Of a number having any given number of decimal orders?

The pupil has seen that the square of any number of tenths is a number of hundredths ; the square of a number of hundredths is a number of ten-thousandths, etc.

Therefore to find the square root of a decimal fraction begin at the decimal point and separate it into periods of two figures each. The square root of the greatest square in the first period to the right of the decimal point is the tenths' figure of the root. The other digits are found as for integers. If there is an odd number of decimal orders annex a zero.

To find the square root of a mixed decimal, such as 333.4276, begin at the decimal point and separate into periods of two figures each. Thus, 3′33.42′76.

EXAMPLE 1. Find the square root of .00498436.

SOLUTION.

$$\begin{array}{r} .00'49'84'36).0706 \\ 49 \\ \hline 1406 \quad 8436 \\ \quad\quad 8436 \\ \hline \end{array}$$

EXAMPLE 2. Find to three decimal places the square root of .4.

SOLUTION.

$$\begin{array}{r} .40'00'00).632 \\ 36 \\ \hline 123 \quad 400 \\ \quad\quad 369 \\ \hline 1262 \quad 3100 \\ \quad\quad 2524 \\ \hline 1264 \quad 57600 \end{array}$$

In this solution it was necessary to find the trial divisor for the fourth figure in order to be sure that this fourth figure is less than 5.

Exercise 32

1. Find the first figure which is not zero in the square root of each of the following : .04 ; .40 ; .4 ; .0009 ; .009 ; .1 ; .03 ; .003 ; .3 ; .16 ; 1.6 ; .016 ; .0016.

Find to the nearest .0001 the square root of each of the following:

2. .0256. 3. 2.563. 4. 2. 5. 3. 6. 5.

7. 1.6. 8. 160. 9. .00341. 10. 1.29.

Reduce these common fractions to decimals, then find their square roots to the nearest .0001.

11. $\frac{1}{2}$. 12. $\frac{1}{3}$. 13. $\frac{1}{4}$. 14. $\frac{3}{7}$.

15. $3\frac{2}{5}$. 16. $6\frac{1}{4}$. 17. $.02\frac{1}{2}$. 18. $.00\frac{1}{3}$.

19. How long is one side of a square whose area is one acre?

20. Find the side of a square whose area is .3 of an acre.

21. A certain rectangle is three times as long as it is wide and its area is 15 sq. ft. What is its width to the nearest .01 ft.?

HINT. Divide the rectangle into squares and find the side of one of the squares.

32. The square root of a common fraction.

Multiply $\frac{3}{5}$ by itself.

Square $\frac{3}{4}$; $\frac{7}{8}$; $\frac{5}{6}$. Make a rule for squaring a fraction.

Find the square root of $\frac{4}{9}$. Check the result by squaring it.

Find the square root of $\frac{64}{121}$. Check the result.

Make a rule for finding the square root of a common fraction.

The rule is easily applied if both the numerator and the denominator are square numbers. In other cases it is simpler to reduce the common fraction to a decimal and then extract the square root.

Rule. *The square root of a common fraction is found by taking the square root of both numerator and denominator.*

Exercise 33

Find the square root of each of the following :

1. $\frac{25}{36}$. **2.** $\frac{49}{64}$. **3.** $\frac{16}{81}$. **4.** $\frac{121}{289}$. **5.** $\frac{256}{441}$.

6. $2\frac{7}{9}$. **7.** $30\frac{1}{4}$. **8.** $69\frac{4}{9}$. **9.** $10\frac{6}{25}$. **10.** $1\frac{17}{64}$.

11. Find in two ways the square root of $\frac{2}{5}$ to four decimal places. First reduce to a decimal, then extract the root; next extract the square root of both terms and reduce the result to a decimal. Which is the simpler way?

12. Find in two ways the square root of $\frac{3}{7}$ to four decimal places.

HINT. First multiply both terms of the fraction by 7.

13. Find in two ways the square root of $\frac{5}{8}$ to four decimal places.

Exercise 34

1. Define square root.

2. State the principle for finding the square of the sum of two numbers.

3. Use this principle in finding the following squares: $(t+w)^2$; $(60+8)^2$; $(7+\frac{1}{2})^2$; $(40+a)^2$.

4. Find the square root of each of the following squares: $t^2+2tw+w^2$; $60^2+2\cdot 60\cdot 8+8^2$; $7^2+2\cdot 7\cdot \frac{1}{2}+(\frac{1}{2})^2$; $40^2+2\cdot 40\cdot a+a^2$.

Find the following correct to .01 :

5. $\sqrt{5929.}$
6. $\sqrt{30276.}$
7. $\sqrt{228484.}$
8. $\sqrt{765625.}$
9. $\sqrt{99.}$
10. $\sqrt{3.1416.}$
11. $\sqrt{842.}$
12. $\sqrt{1.7.}$
13. $\sqrt{\frac{16}{25}.}$
14. $\sqrt{\frac{2}{3}.}$
15. $\sqrt{\frac{1}{7}.}$
16. $\sqrt{35\frac{1}{8}.}$

Name the first figure which is not zero in the square root of each of the following :

17. .1.
18. .01.
19. .001.
20. .0001.
21. .004.
22. .00016.
23. .03.
24. .9.
25. .000032.
26. .0005417.
27. 2.5.
28. .7923.

29. Find correct to .0001, the side of a square whose area is 2 sq. ft.

30. The area of one face of a cube is .0006 m^2. Find the length of one edge of the cube correct to .001m.

31. It is shown in geometry that the area, A, of a triangle whose sides are a, b, and c, is given by the formula

$$A=\sqrt{S(S-a)(S-b)(S-c)},$$

where $S=\frac{a+b+c}{2}$. Find the area of a triangle whose sides are 14, 10, and 8.

32. Find the number of acres in a triangular field whose sides are 18 rd., 28 rd., and 32 rd.

33. Relation between the sides and the hypotenuse of a right triangle. The side of a right triangle opposite the right angle is called the **hypotenuse.** The other two sides are usually referred to as the **sides.**

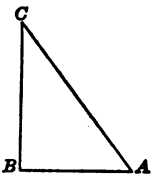


FIG. 10

In Figure 10, AB is the base and BC is the altitude of the right triangle.

Construct a right triangle with sides 3 in. and 4 in. Measure the hypotenuse. If your measurement and construction are accurate the hypotenuse will be found to be 5 in.

Construct a right triangle with sides 6 in. and 8 in. Measure the hypotenuse. How long is it?

Construct another right triangle with sides 5 in. and 12 in. How long is the hypotenuse?

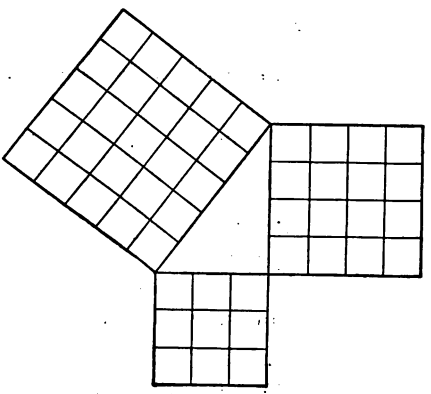


FIG. 11

Construct squares on the three sides of a right triangle whose sides are 3 in., 4 in., and 5 in., as in Figure 11. Since $9+16=25$, the square on the hypotenuse of this right triangle equals the sum of the squares on the other two sides.

Test to see if this is true for the other two triangles that you have constructed. These examples illustrate the following important fact known as the

Pythagorean theorem. *The square on the hypotenuse of a right triangle equals the sum of the squares on the other two sides.*

It is proved in geometry that this theorem is true for all right triangles. It is one of the most important theorems

in geometry and is used in solving many practical problems.

If h is the hypotenuse, a the altitude, and b the base of a right triangle as in Figure 10, then the Pythagorean theorem may be stated by the

Formula $h^2=a^2+b^2$.

By taking the square root of each side of this equation we get

$$h=\sqrt{a^2+b^2}.$$

This gives a formula for finding the hypotenuse of a right triangle when the two sides are known. Let the pupil state this formula as a rule.

Exercise 35

Find the hypotenuse of a right triangle, given the sides as follows :

1. 15 in., 20 in.
2. 12 in., 35 in.
3. 10 in., 10 in.
4. 100 ft., 65 ft.

5. From the formula $h^2=a^2+b^2$ we wish to get a formula to find the altitude of a right triangle when the base and the hypotenuse are known. From $h^2=a^2+b^2$ find a^2. Now find a. State as a rule the formula you have found.

6. Obtain a formula for finding the base of a right triangle when the hypotenuse and the altitude are given. State this formula as a rule.

7. Find a if $h=50$ ft. and $b=14$ ft.

8. Find b if $a=21$ and $h=35$.

9. Find the base of a right triangle whose altitude is 65 ft., and hypotenuse 98 ft.

10. Find the altitude of a right triangle whose hypotenuse is 56.56 rods, and base 40 rods.

11. The sides of a right triangle are 15 in. and 39 in. What is the sum of the squares on these sides? What is the square on the hypotenuse? How long is the hypotenuse?

12. Find the hypotenuse of a right triangle whose sides are 12 and 26.

13. Find the diagonal of a rectangle whose dimensions are 20 in. and 30 in.

14. A baseball diamond is a square 90 ft. on a side. The catcher stands 3 ft. behind home plate and throws to second base. How far does he throw?

15. What is the longest straight line that can be drawn on a piece of paper 8 in. by 11 in.?

16. The hypotenuse of a right triangle is 60 ft. and one side is 36 ft. What is the square on the hypotenuse? What is the square on the given side? What is the square on the other side? Find the length of the other side.

17. A rope 100 ft. long is attached to the top of a flagpole and reaches to a point on the ground 80 ft. from the foot of the pole. How high is the pole?

18. A ladder 20 ft. long is set with its foot 8 ft. from the base of a vertical wall and with its top resting against the wall. How high up the wall does it reach?

19. Olive and Helen wish to cross the streets from A to C. Olive "cuts the corner" from A to C, and Helen crosses from A to B and then from B to C. (Figure 12.) How much farther does Helen walk than Olive?

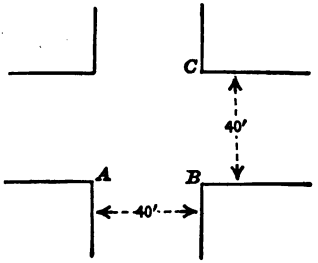


FIG. 12

20. A house is 18 ft. wide and the ridgepole is 9 ft. above the plate. Find the length of the rafter if it reaches 1 ft. beyond the plate.

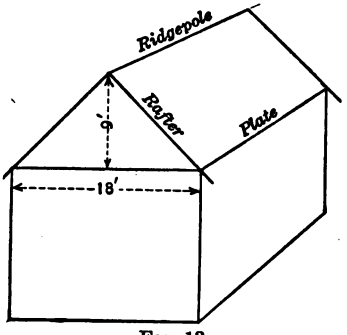


FIG. 13

21. Two fence posts each 4 ft. high are 8 ft. apart. How long must a brace be made to reach from the bottom of one post to a point 6 in. below the top of the other?

22. A kite string is 400 ft. long. The boy who holds one end of the string is 200 ft. from a point directly under the kite. How high is the kite? Make no allowance for slack in the string.

23. The pitcher in a baseball game catches a batted ball while standing at the point of intersection of the diagonals of the diamond. The batter and the pitcher both run to first base. How much farther does the batter have to run than the pitcher?

24. A room is 30 ft. long, 20 ft. wide, and 12 ft. high. Find the length of a wire that reaches from one upper corner to the opposite lower corner.

25. How many times the side is the diagonal of a 1-inch square? Answer the same question for a 3-inch square; for a 20-inch square; for a square whose side is a.

26. By what must the side of a square be multiplied to get the diagonal? If given the diagonal, how can you find the side? Make formulas which give the answers to these questions, using d for the diagonal and s for the side.

27. The side of an equilateral triangle ABC, Figure 14, is 10 in. Find the altitude, knowing that D, the foot of the altitude, bisects the base AB. Find the answer correct to

.001 sq. in. Show that the altitude of this equilateral triangle is .866s, where s is the side. Find the altitude of an equilateral triangle whose side is 100 ft.; 30 ft.; 80 rods.

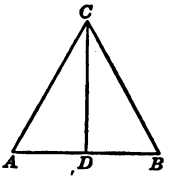


FIG. 14

28. The last exercise gives the formula $h = .866s$, where h is the altitude of an equilateral triangle and s is the side. Find the altitude of an equilateral triangle whose side is 450 ft.; 65 ft.; 320 rods.

29. How can you find the side of an equilateral triangle if you know the altitude? State your answer as a formula. Find the side of an equilateral triangle whose diagonal is 173.2 ft.; 13.856 ft.; 26 rods.

30. Find the area of an equilateral triangle whose side is 10 in.

31. Explain how this formula for the area of an equilateral triangle is made: $A = \frac{hs}{2} = .866\, s \times \frac{s}{2} = .433\, s^2$. Here A represents the area, h the altitude, and s the side.

32. Find the area of an equilateral triangle whose side is 16 in.

33. Show that a regular hexagon is made up of six equilateral triangles. Find the area of a regular hexagon whose side is 4.5 in.

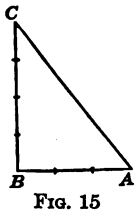


FIG. 15

34. Finding square roots graphically. In the right triangle ABC (Figure 15), $AB = 3$, $BC = 4$, and therefore $AC = 5$. The length of AC may be tested by using the compasses. Open the compasses the distance AC and see if this distance is 5.

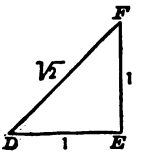


FIG. 16

In the right triangle DEF, $DE = 1$, $EF = 1$, and therefore $DF = \sqrt{2}$. If $DE = a$ and $EF = a$, what is the length of DF?

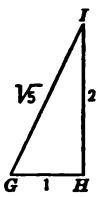


FIG. 17

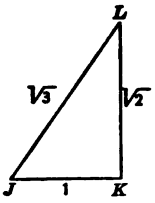


FIG. 18

Suppose that we wish to construct a line of length $\sqrt{5}$. This is done in the right triangle GHI. Tell how this construction may be made.

In triangle JKL the line $LJ = \sqrt{3}$. Tell how this construction may be made.

Exercise 36

1. Construct the square root of 7. See Figure 19.

2. Construct the square root of 13. Notice that $13 = 9 + 4$.

3. Construct the square root of 20; of 10; of 41; of 61. Check the answers by measuring with the compasses.

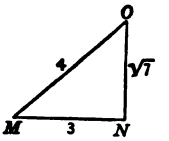


FIG. 19

4. If the hypotenuse of a right triangle is 6 and the base 3, what is the altitude? Make such a triangle and thus construct $\sqrt{27}$.

5. Construct $\sqrt{45}$.

HINT. Make the hypotenuse 7. What then shall the base be made?

6. I construct a square with side 10. I then construct a second square with a side equal to the diagonal of the first. What is the area of the first square? Of the second square? The area of the second square is how many times the area of the first?

7. I construct a square with side a. I then construct a second square with a side equal to the diagonal of the first. What is the area of the first square? Of the second square? The area of the second square is how many times the area of the first?

8. I wish to construct a square with an area twice as large as the area of a given square. How should I proceed?

9. A gardener wishes to lay out a square flower bed twice as large as another square one whose side is 16 ft. Can you tell him how long the side of the new bed should be made?

10. The distance that a sailor can see from the top of a mast is given approximately by the formula $d=1.22\sqrt{h}$, where d is the distance in miles and h is the height in feet of the mast above the water. How far can a sailor see if he is 30 ft. above the water? If he is 80 ft. above the water?

11. The velocity v of the water at the bottom of a stream is calculated by the formula $v=V+1-2\sqrt{V}$, where V is the velocity of the surface. Find v if V is 2 miles an hour ; if V is 100 ft. a minute.

12. $T=2\pi\sqrt{\frac{l}{g}}$ Find T if $l=36$ and $g=32.2$.

13. If a body falls h feet, its velocity, V, is given by the formula $V=\sqrt{2gh}$, where $g=32.2$. In this formula the resistance of the atmosphere is neglected. Find the velocity when it reaches the ground of a ball dropped from a tower 90 ft. high.

14. A pile driver falls 18 ft. How fast is it going when it strikes?

15. A hickory nut falls 40 ft. How fast is it going when it strikes the ground?

CHAPTER IV

RATIO, PROPORTION, AND SIMILAR FIGURES

35. Ratio. The **ratio** of one number to another of the same kind is the quotient of the first divided by the second.

The ratio of 3 to 4 is $\frac{3}{4}$. It is also written 3 : 4.

The ratio of a to b is written $\frac{a}{b}$, or $a : b$. The numbers a and b are called the **terms** of the ratio.

The ratio of \$10 to \$12 is $\frac{10}{12}=\frac{5}{6}$. The ratio of 8 ft. to 6 in. is $\frac{96 \text{ in.}}{6 \text{ in.}}=16$.

The quotient $\$10 \div 2$ is not a ratio since the dividend and the divisor are not like numbers.

It is seen that any fraction expresses a ratio. A ratio is an abstract number.

Exercise 37

In the following find the ratio of the first number to the second. Give the results in simplest form.

1. 16, 4.	6. 5, 7.	11. $\frac{1}{2}$, 2.	16. .02, .0008.
2. 4, 16.	7. 45, 80.	12. 7, $\frac{1}{6}$.	17. 568, .0004.
3. 9, 3.	8. 134, 48.	13. $\frac{1}{2}$, $\frac{1}{3}$.	18. 6 in., 8 in.
4. 3, 9.	9. 0, 5.	14. $\frac{5}{6}$, $\frac{4}{9}$.	19. 2 ft., 2 in.
5. 17, 17.	10. 0, $\frac{1}{2}$.	15. .7, 1.05.	20. \$100, 5¢.
21. 1 gal., 1 cu. ft.	25. a ft., m yd.	29. 2×5, 3×5.	
22. 30′, 5 degrees.	26. m, n.	30. $2a$, $3a$.	
23. 1 bu., 1 gal.	27. 2, x.	31. 5, 5.	
24. a ft., m ft.	28. x, 14.	32. a, a.	

33. $4\times6\times7$, $4\times6\times7\times9$.

34. 0, .0001.

35. 1 mile, 1 ft.

36. 10%, 14%.

37. .1%, 4%.

38. ab, ac.

39. $3a^2$, $2a$.

40. $\frac{2}{3}a$, $.01a$.

41. What is the ratio of \$16 to \$40? Write the answer as a common fraction, as a decimal fraction, and as a per cent.

42. What is the ratio of \$10 to \$20? \$10 is what part of \$20? What per cent?

43. What is the ratio of one side of a square to the perimeter of the square? One side is what part of the perimeter? One side is what per cent of the perimeter?

44. What is the ratio of the circumference of a circle to the diameter? The circumference is how many times the diameter? The circumference is what per cent of the diameter?

45. The radii of two circles are 6 in. and 8 in. What is the ratio of the radii? Of the diameters? Of the circumferences?

46. Answer the same questions as in the preceding exercise if the two radii are a and b.

47. A map is drawn on the scale of 1 in. to 100 miles. What is the ratio of lines on the map to the corresponding distances on the earth's surface?

48. One side of a square is 3 in. Find its diagonal, then find the ratio of the diagonal to one side, correct to 3 decimal places. Find the square root of 2 to 3 decimal places.

49. One side of a square is 5 in. Find the ratio of the diagonal of the square to one side.

50. One side of a square is 10. Find the ratio of its diagonal to one side.

51. One side of an equilateral triangle is 2. Find the ratio of its altitude to one side.

36. Specific gravity. In many commercial and mechanical problems it is necessary to know how to find the weights of articles of large sizes which cannot easily be weighed but whose volumes are known. How can an ice man determine how many tons of ice are in his ice house? Ships are made of concrete and steel. How can the weights of the materials be found if the volumes are known? The chemist and the physicist must be able to compute the weights of many substances whose volumes are known.

There are many problems of this kind. So it becomes necessary to make tables by the use of which such weights can be found. Such tables usually do not give the weights of the substances directly, but tell how many times as heavy as water each substance is. Thus gold is 19.28 times as heavy as water and ice is .92 as heavy as water. To say that gold is 19.28 times as heavy as water means that a certain volume of gold weighs 19.28 times as much as the *same volume* of water. For example, one cubic foot of gold weighs 19.28 times as much as one cubic foot of water.

Preliminary Questions

1. A brick weighs 4 lb. A piece of wood of the same size weighs 1 lb. The brick is how many times as heavy as the piece of wood? What is the ratio of the weight of the piece of wood to the weight of the brick?

2. A piece of wood weighs 2.4 lb. The same number of cubic inches of water weighs 3 lb. What is the ratio of the weight of the wood to the weight of the water?

3. A cubic foot of water weighs 1000 oz. A cubic foot of oak weighs 700 oz. What is the ratio of the weight of oak to the weight of an equal volume of water?

4. A cubic foot of tin weighs 7300 oz. What is the ratio of the weight of tin to the weight of an equal volume of water?

5. A cubic foot of dry sand weighs 1400 oz. What is the ratio of the weight of sand to the weight of an equal volume of water?

6. Clay is 1.9 times as heavy as water. What is the weight of 1 cu. ft. of clay?

7. Cast iron is 7.2 times as heavy as water. How much does a cubic foot of cast iron weigh? A cubic inch of cast iron?

8. Since ice is .92 times as heavy as water, how much does a piece of ice $2' \times 8'' \times 27''$ weigh?

9. Mercury is 13.57 times as heavy as water. How much does a cubic inch of mercury weigh?

10. Cork is .24 as heavy as water. Find the weight of a piece of cork $2' \times 1'6'' \times 3''$.

11. Find the weight of a block of metal $4'' \times 3'' \times 3''$, the metal being 18 times as heavy as cork.

One cubic foot of silver weighs 10.47 times as much as a cubic foot of water; that is, the ratio of the weight of one cubic foot of silver to the weight of one cubic foot of water is 10.47. Silver is said to have a **specific gravity** of 10.47 compared with water.

The ratio of the weight of a gallon of alcohol to the weight of a gallon of water is .84. Alcohol is said to have a specific gravity of .84 compared with water.

The **specific gravity** of a substance compared with water as a standard is the ratio of the weight of a given volume of the substance to the weight of an equal volume of water.

In question 2 above what is the specific gravity of the piece of wood? In question 3 what is the specific gravity of oak? What is the specific gravity of tin? Of dry sand? Of clay?

The following table gives the specific gravities of certain substances compared with water.

TABLE OF SPECIFIC GRAVITIES

SUBSTANCE	SPECIFIC GRAVITY	SUBSTANCE	SPECIFIC GRAVITY	SUBSTANCE	SPECIFIC GRAVITY
Gold . .	19.28	Cast iron . . .	7.2	Cork . .	.24
Platinum	21.50	Bituminous coal	1.4	Sea water	1.03
Mercury	13.57	Ebony	1.33	Alcohol .	.84
Steel . .	7.83	Walnut . . .	.67	Ice . .	.92

Exercise 38

In the following exercises find the answers correct to two decimal places:

1. A cubic foot of a certain substance weighs 3 times as much as a cubic foot of water. What is the specific gravity of the substance?

2. A cubic foot of water weighs 62.5 pounds. What is the weight of a cubic foot of ice? How many cubic feet of ice are there in 1 ton?

3. The ice man leaves a piece of ice containing $1\frac{3}{4}$ cubic feet. By how much does the weight of this piece differ from 100 lb.?

4. The weight of a quart of alcohol is what part of the weight of a quart of water?

5. A gallon of sea water weighs how many times as much as a gallon of pure water?

6. A cubic foot of sulphur weighs 125 pounds. What is the specific gravity of sulphur?

7. A cubic foot of flint glass weighs 187.5 pounds. What is the specific gravity of flint glass?

8. If the specific gravity of pure milk is taken as 1.032, what does a gallon of milk weigh?

9. How many cubic feet of bituminous coal weigh a ton?

10. Will a piece of steel float in mercury? Will a piece of gold float in mercury? Will ebony float in water? Will ice float in alcohol?

11. If alcohol and olive oil, specific gravity .91, were put into the same vessel, which would go to the bottom?

12. A cylindrical piece of cork 1 in. in diameter and 2 in. long weighs how many times as much as an equal volume of water?

13. How many cubic inches of water weigh a pound? Is it approximately true for a pint of water, that "a pint is a pound the world around"?

14. Express the specific gravities in the table as per cents.

15. What is the weight of a cubic centimeter of water in metric units? Of a cubic meter of water? Of a liter? (See the metric tables.)

16. What is the weight of a liter of alcohol? Of a liter of sea water? Of a cubic meter of coal? Of a cubic meter of steel?

17. What is the volume of 15 grams of water? Of 15 grams of cork?

18. What is the volume of 250 kg. of each of the substances in the table of specific gravities?

19. A kilogram of platinum is how many times as large as a kilogram of cast iron?

20. A load of bituminous coal weighs 4200 lb. How many cubic feet of coal in the load? Use the result of exercise 9.

21. A stick of walnut timber contains 12.3 cu. ft. How much does it weigh?

22. The ice man hauls 60 cu. ft. of ice at a load. How much does the load weigh?

23. A piece of steel has a volume of 320 cu. in. How much does it weigh?

37. Dividing a number into parts having a given ratio.

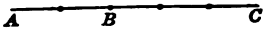


FIG. 20

What is the ratio of AB to BC? Into how many equal parts is AC divided?

AB is what part of AC?

BC is what part of AC?

It is seen that the line AC is divided into two parts which have the ratio of 2 : 3.

If it is desired to divide the line MN (Figure 21) into two parts having the ratio of 3 : 4, into how many equal parts must the line MN be divided?

How many of these equal parts in the first part of MN? In the second part?

FIG. 21

EXAMPLE. Divide 84¢ into two parts which have the ratio of 3 : 4.

SOLUTION. Since the two parts of 84¢ are to have the ratio of 3 to 4, we divide the 84¢ into 7 equal parts. For the first of the two parts we take 3 of the equal parts and for the second we take 4 of the equal parts.

$\frac{3}{7}$ of $84¢ = 36¢$, one of the required parts.

$\frac{4}{7}$ of $84¢ = 48¢$, the second required part.

CHECK. $\frac{36¢}{48¢} = \frac{3}{4}$, and $36¢ + 48¢ = 84¢$.

Exercise 39

1. Divide \$65 into two parts having the ratio of 5 : 8.

2. Two boys have 30 marbles which they wish to divide in the ratio of 2 : 3. How many should each receive?

3. A man and a boy, working together, receive \$30 for a piece of work. They agree to divide that amount in the ratio of 5 to 3. How much does each receive? The man gets how many times as much as the boy?

4. The shares received by two heirs to an estate are in the ratio of 2 to 5. Their combined shares are \$16,000. Find the share of each.

5. The seventh and the eighth grades have a picnic together. There are 47 pupils in the seventh grade and 62 in the eighth. The total expenses are $26.48. How much should each grade pay?

6. A mixture of water and alcohol contains 1 part of alcohol to 3 parts of water. How much of each in 12 ounces of the mixture?

7. Concrete blocks are made of 1 part of cement to 3 parts of sand. How much of each in 240 cu. ft. of concrete blocks?

8. A concrete mixture for large engine foundations contains 1 part cement, 2 parts sand, and 4 parts crushed stone. How much of each in a foundation 12 ft. wide, 20 ft. long, and 4 ft. deep?

9. Two men engage in business together. The first invests $5000 and the second $7000. What is the ratio of the investment of each to the whole investment? The profits for the first year are $1500. They are divided in the ratio of the investments. How much does each man receive?

10. A drainage ditch costs $3875. The expense is borne by four farmers according to the number of acres of land that is drained for each. The ditch drains 200 acres for A, 180 acres for B, 400 for C, and 80 for D. How much should each pay?

11. Two men ship their hogs to market in the same car. The freight and other charges are $119.74. A's hogs weigh 3540 pounds and B's weigh 6085 pounds. What is the share of each in the expenses?

12. Two boys work at a job and agree to share the pay according to the number of hours each works. One works 12 hours and the other 15 hours. What part of the pay should each receive?

13. A certain recipe for milk sherbet is as follows: Milk, 4 parts; sugar, $1\frac{1}{2}$ parts ; lemon juice, $\frac{1}{2}$ part. How much of each should be used to make 2 gallons of the sherbet?

38. Proportion. The statement of the equality of two ratios is called a **proportion.**

Thus, the statement $\frac{5}{6}=\frac{10}{12}$ is a **proportion.**

The proportion $\frac{a}{b}=\frac{c}{d}$ may also be written

$$a:b=c:d.$$

In either case the proportion may be read "a is to b as c is to d," or "the ratio of a to b equals the ratio of c to d."

In the proportion $a:b=c:d$, a, b, c, and d, are called the **terms** of the proportion; a and d are called the **extremes** and b and c are the **means.**

The four numbers which form a proportion are said to be proportional.

Principle. *In any proportion the product of the means equals the product of the extremes.*

If $\frac{a}{b}=\frac{c}{d}$, both members of the equation may be multiplied by bd.

Then $ad=bc$.

This principle gives a convenient way to find one term of a proportion when the other three terms are given.

EXAMPLE 1. Find x, if $x:10=2:5$.

SOLUTION. $5x=20$, by the principle just given.
Then $x=4$.

CHECK. $4:10=2:5$.

EXAMPLE 2. Find x if $9:20=x:12$.

SOLUTION. $20x=108$. The product of the means equals the product of the extremes.
$x=5\frac{2}{5}$, dividing each member by 20.

CHECK. $9:20=5\frac{2}{5}:12$, or $\frac{9}{20}=\frac{5\frac{2}{5}}{12}=\frac{27}{60}$.

Exercise 40

Solve each of the following proportions for the letter involved :

1. $x : 2 = 7 : 3$.
2. $x : 9 = 5 : 3$.
3. $6 : x = 18 : 30$.
4. $7 : 40 = x : 120$.
5. $8 : 11 = 24 : x$.
6. $12 : a = 6 : 19$.
7. $30 : 5 = a : 11$.
8. $\frac{x}{9} = \frac{5}{3}$.
9. $\frac{x}{16} = \frac{13}{8}$.
10. $\frac{45}{m} = \frac{15}{36}$.
11. $m : .2 = 5 : 8$.
12. $m : .5 = 1.2 : .01$.
13. $6.5 : .13 = 84 : m$.
14. $a : \frac{2}{3} = \frac{6}{7} : 5$.
15. $a : \frac{1}{5} = \frac{4}{5} : 12$.

39. Proportional lines. Four lines are said to be proportional when the ratio of the first to the second equals the ratio of the third to the fourth.

Draw a triangle ABC with $AB = 6$ in., $BC = 4$ in., and $AC = 3$ in.

Take point H on AB, $1\frac{1}{2}$ in. from A, and draw HK parallel to AC.

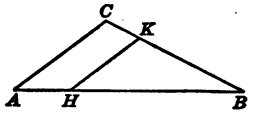


FIG. 22

With the ruler measure BK and KC.

What is the ratio of AH to HB? Of CK to KB?

Compare these ratios. If your construction and measurements were accurate this statement can be made as a proportion. Make it.

Locate point P on AB so that AB is divided in the ratio of 1 : 7.

How long is AP? PB?

Through P draw PQ parallel to AC.

Measure BQ and QC. Compare $\frac{AP}{PB}$ and $\frac{BQ}{QC}$, making the statement as a proportion.

Draw a triangle XYZ and divide one side into segments having the ratio 3 : 5. Through the point of division draw a line parallel to another side of the triangle. This line

divides the third side of the triangle into two segments. What is their ratio?

In every case the pupil will find that if a line be drawn parallel to one side of a triangle, it divides the other two sides into four segments which are proportional. This is briefly stated in the following

Principle. *A line parallel to one side of a triangle divides the other two sides proportionally.*

When a line is drawn parallel to one side of a triangle, this principle may be used to find any one of the four segments of the other two sides if three of the segments are known. Thus in Figure 22, if AH is $1\frac{1}{2}$ in., HB is $4\frac{1}{2}$ in., and CK is $1\frac{1}{3}$ in., we may write the proportion $\frac{4\frac{1}{2}}{1\frac{1}{2}} = \frac{x}{1\frac{1}{3}}$, in which x is BK.

Solving this proportion, $x = 4$, the length of BK.

In Figure 22 there are other interesting ratios which help in finding the lengths of other lines in the figure. Measure the length of HK.

Compare the ratios $\frac{AB}{HB}$, $\frac{CB}{KB}$, and $\frac{AC}{HK}$. Make these statements as three proportions. Are they correct? If not you have made a mistake in drawing or measuring.

Compare the ratios $\frac{AB}{AH}$ and $\frac{CB}{CK}$.

The pupil should draw other triangles with lines parallel to the sides and select the equal ratios.

PROBLEM. In Figure 22, knowing that $AB = 6$ in., $BC = 4$ in., $AC = 3$ in., and $HB = 4\frac{1}{2}$ in., find the length of HK.

SOLUTION. Let $y = HK$.

Then $$\frac{AB}{HB} = \frac{AC}{HK}, \text{ or } 6 : \frac{9}{2} = 3 : y.$$

$$6y = \tfrac{27}{2}.$$

$$y = \tfrac{9}{4}.$$

Care must be taken to choose corresponding segments for the first and third terms, also for the second and fourth.

Exercise 41

1. Draw a triangle ABC and line HK parallel to AB, cutting AC at H and BC at K. Complete these proportions: $AH : HC = BK : ?$; $BC : BK = ? : AH$; $AC : HC = AB : ?$.

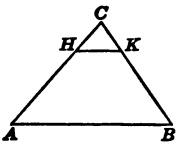


FIG. 23

2. In triangle PRS, MN is drawn through M, the midpoint of PR, parallel to RS. PM is what part of PR? PN is what part of PS? MN is what part of RS?

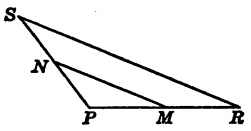


FIG. 24

What is the ratio of PR to PM? Of MN to RS?

What per cent of PS is NS?

3. To find the distance across a pond two boys run lines BC and AC. They then find their mid-points R and S. They run line RS and measure it. They then compute the length of AB. How? If they find RS to be 120 ft., how long is AB?

HINT. BC is how many times SC? Then AB is how many times RS?

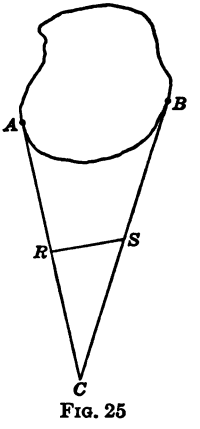


FIG. 25

4. Use the plan of the preceding exercise to compute the length of a sheet of paper. Check your result by measuring its length.

5. The top of a ladder reaches a point on a wall 20 ft. from the ground. How long should an upright support of the ladder be if placed halfway between the foot of the ladder and the wall? If placed $\frac{2}{3}$ of the distance from the wall to the foot of the ladder? If placed $\frac{2}{3}$ of the distance from the foot of the ladder to the wall?

6. Draw a triangle having side $AB = 4$ in., $AC = 3$ in., $BC = 6$ in. Take point P on BC $\frac{3}{4}$ in. from B. Draw PT parallel to CA. Complete these proportions: $BP : PC = BT :$; $BC : BP = BA :$; $BP : BC =$ $: CA$.

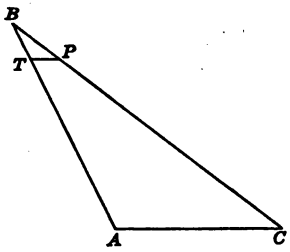


FIG. 26

Substitute the values for the given lengths and use single letters for those not given. Solve the proportions for these letters.

40. Similar figures. When a drawing is made to scale or when a picture is enlarged, the resulting figure has the same shape as the original.

Figures that have the same shape are called **similar figures.**

Corresponding parts of similar figures are those that occupy similar positions. Thus in Figures 27 and 28 AB and $A'B'$ are corresponding sides, and $\angle A$ and $\angle B$ are corresponding angles. In Figures 30 and 31 CD and $C'D'$ are corresponding sides and $\angle D$ and $\angle D'$ are corresponding angles.

Measure the sides of Figure 27 and Figure 28 and find the ratio of each side of Figure 28 to the corresponding side of Figure 27. What is true of these ratios? Do the same for Figure 30 and Figure 31. What is true for these ratios?

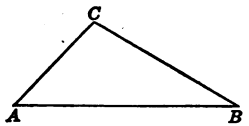


FIG. 27

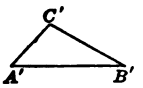


FIG. 28

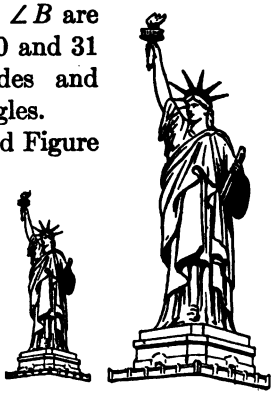


FIG. 29

Measure some of the corresponding angles in these figures. What do you find to be true of the corresponding angles?

These facts suggest the following

Definition. *Similar figures are figures whose corresponding sides are proportional and whose corresponding angles are equal.*

Similar triangles. Of the various kinds of similar figures we shall make most use of similar triangles.

Draw two triangles having two angles of one equal respectively to two angles of the other. Are the third angles equal? Why?

Measure the sides of these triangles. What must be shown to prove that the triangles are similar? Are they similar?

This illustrates the

Principle. *If two angles of one triangle are equal to two angles of another, the two triangles are similar.*

Exercise 42

1. Read the corresponding sides and the corresponding angles in Figures 30 and 31.

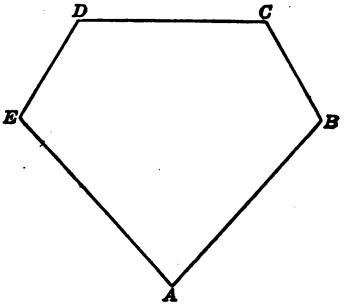


FIG. 30

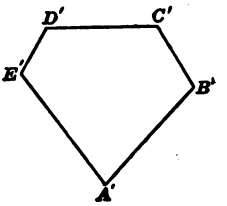


FIG. 31

What is true of the corresponding angles? Of the corresponding sides?

2. Are all squares similar?

3. Make two rectangles. Make the base of one 6 in. and the altitude 4 in., and the base of the other 3 in. and its altitude 1 in. Are these two rectangles similar? Are their corresponding angles equal? Are their corresponding sides proportional? What is the ratio of their bases? Of their altitudes?

4. Are all rectangles similar?

5. Are all equilateral triangles similar?

6. What must be shown to be true of two figures to show that they are similar?

7. Do all similar figures have the same area?

8. Are all triangles which have the same area similar? Illustrate.

9. Two circles have radii of 4 in. and 7 in. respectively. What is the ratio of their radii? Of their diameters? Of their circumferences?

10. These two triangles, Figures 32 and 33, are similar. Then $x : 10 = 20 : 12$. Why? Find the value of x; also of y.

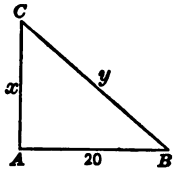


FIG. 32

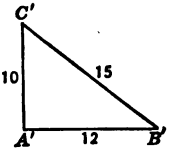


FIG. 33

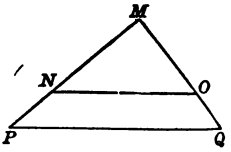


FIG. 34

11. In Figure 34 NO is parallel to PQ. In the triangles MNO and MPQ, what angles are equal? Are the triangles similar? Why? If $MN = \frac{2}{3} MP$, NO is what part of PQ? MO is what part of MQ?

12. A boy, wishing to know how high his kite was, tied the kite string to a peg in the ground at A, cut a pole ten feet long and held it upright at B, where its top just touched the string at P. He then measured the line AP and found it to be 18 ft. He knew the length of the kite string to be 400 ft. Assuming the string to be stretched tight, he then computed the height of the kite. How high was it?

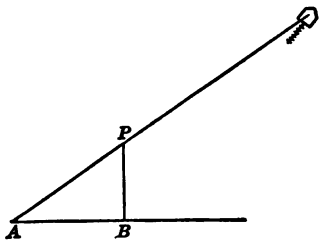


FIG. 35

13. A ten-foot pole standing upright casts a shadow 6 ft. long at the same time that a tower casts a shadow 90 ft. long. How high is the tower? Illustrate by a diagram.

14. A 20-foot ladder rests with its foot 12 ft. from a wall and its top 16 ft. from the ground. How high from the ground is a boy if he has climbed up 5 ft. of the ladder? If he has climbed up 10 ft.? 15 ft.? 18 ft.? How far from the wall is he at each of these points? Make a diagram.

15. A boy lies on the ground with his feet against the foot of an upright pole and sights at the top of a tree. The pole is 10 ft. long. The distance of the boy's eye from the foot of the pole is 5 ft. 2 in., and from the foot of the tree is 47 ft. 8 in. Find the height of the tree.

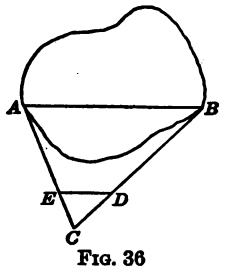


FIG. 36

16. It is desired to find the distance AB across a pond. From a point C the distances AC and BC are measured and are found to be 212 ft. and 276 ft. respectively. CD is then made $\frac{1}{4}$ as long as BC and CE $\frac{1}{4}$ as long as AC. The triangles ABC and CDE are then similar. DE is then measured and found to be 70 ft. 6 in. Find AB.

17. Make a plan for finding the distance across a river when it is not possible actually to measure that distance. (Figure 37.)

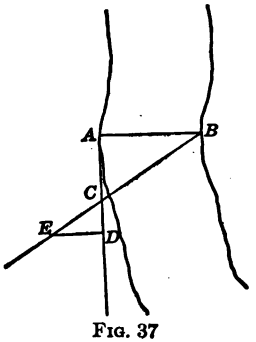


FIG. 37

18. Measure the diameter of a dime. How far from the eye must a dime be held so as just to hide the full moon, if the moon's diameter is 2163 miles and its distance from the earth is 238,840 miles?

19. A boy holds a pencil which is 5 in. long in a vertical position with the lower end of the pencil on a level with his eye, and 2 ft. from his eye. In that position the top of the pencil is in line with the top of a tree which is 80 ft. away. The distance from the boy's eye to the ground is 4 ft. 8 in. How high is the tree?

20. Use the method of the above problem to find the heights of objects.

21. Show how a similar method might be used to find the distance to an airplane if the length of the airplane is known.

22. In this figure we wish to find the distance xy from the shore of a lake to the shore of an island. What measurements need to be made to find the distance?


FIG. 38

41. Ratio of areas of similar figures.

Preliminary Exercises

1. The side of one square is 5 in. and of another 7 in. What is the ratio of their sides? Of their areas? Answer the same questions for two squares whose sides are 3 in. and 5 in.; 10 in. and 13 in.; a in. and b in.

2. If you know the sides of two squares, how can you find the ratio of their areas?

3. The base of a rectangle is 10 in. and its altitude is 6 in. The base of a similar rectangle is 15 in. What is its altitude? What is the ratio of corresponding sides? Of the areas? Reduce these ratios to lowest terms. What relation exists between the ratios?

4. If you know the corresponding sides of two similar rectangles, how can you find the ratio of their areas? What is the ratio of the areas of two similar rectangles which have corresponding sides of 8 in. and 15 in.?

5. The radius of one circle is 8 in. and of another 11 in. What is the ratio of the radii? The area of each is how many times π? What is the ratio of the areas? Answer the same questions if the radii are 7 in. and 10 in.; a in. and b in.

6. If you know the radii of two circles, how can you find the ratio of their areas?

These exercises illustrate the following

Principle. *The areas of similar surfaces are to each other as the squares of corresponding lines.*

Exercise 43

1. The bases of two similar rectangles are 12 ft. and 6 ft. What is the ratio of their areas?

2. Find the ratio of the areas of circles which have radii as follows: 3 in. and 1 in.; 1 in. and 3 in.; 4 in. and 8 in.; 6 in. and 10 in.; 2 in. and y in.

3. If the radii of two circles have the ratio 1 : 2, what is the ratio of their areas? If the radii have the ratio of 1 : 3? 2 : 5? 4 : 1?

4. By what is the area of a square multiplied if the side is multiplied by 2? If the side is multiplied by 3? By 10?

5. The area of a circle 4 in. in diameter is how many times the area of one 2 in. in diameter? The area of one 5 in. in diameter is how many times the area of one 2 in. in diameter?

6. Two similar triangles have bases 12 in. and 15 in. respectively. The area of the first is 48 sq. in. What is the area of the second?

HINT: By using the principle we have $12^2 : 15^2 = 48 : x$, where x is the area of the second triangle.

7. Two similar triangles have altitudes 18 in. and 26 in. respectively. The area of the first is 108 sq. in. What is the area of the second?

8. If cookies 3 in. in diameter cost 15 cents a dozen, what should cookies of the same thickness and 4 in. in diameter cost?

9. If 15 cookies with a two-inch radius weigh a pound, how many of the same thickness and a one-inch radius weigh a pound?

10. A map of Illinois is drawn on a scale of 10 mi. to 1 inch. The area of the State is 56,650 square miles. What is the area of the map? On this map a certain county is $2\frac{1}{4}$ in. by $2\frac{7}{32}$ in. What are its dimensions? What is its area?

11. The floor plan of a house is drawn on the scale of 16′ to 1″. The area of the plan is 8 sq. in. What is the area of the floor?

12. The side of a 10-inch square is increased 20%. By what per cent is the perimeter increased? The diagonal? The area?

13. The radius of a circle is 6 in. The radius is increased $87\frac{1}{2}\%$. By what per cent is the diameter increased? The circumference? The area? It is possible to find these answers without finding the diameter, circumference, and area.

14. A photograph 3 inches wide and 5 inches long is enlarged into one that is $7\frac{1}{2}$ inches wide. How long is it? The area of the enlarged photograph is how many times that of the original? A figure $\frac{5}{8}$ in. long on the original is how long on the enlarged photograph?

15. By what must the side of a square be multiplied to double the area?

16. Draw a triangle. With ruler and compasses construct a triangle similar to the first and having four times the area.

17. Construct a triangle with sides 1 in., $\frac{1}{2}$ in., and $\frac{3}{4}$ in. Construct a triangle similar to the first and having nine times the area.

Exercise 44. Drawing to Scale

1. A map of Illinois is drawn on the scale of 10 miles to 1 inch. The map is 3 ft. $\frac{1}{4}$ in. long. How long is the state?

2. On this map it is 1 ft. $5\frac{1}{8}$ in. from Chicago to Springfield. How many miles from one city to the other?

3. What scale should be used to get a map of Illinois 8 in. long?

4. To find the height of a school tower a line AB, 100 ft. long, was measured; $\angle DAC$ was measured and found to be 27°, and $\angle DBC$, 44°. Make a drawing of this figure on the scale of 40 ft. to one inch and find by measurement the height of the tower.

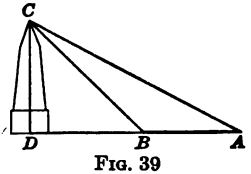


FIG. 39

5. To check the results in the previous exercise the pupils made other measurements. In the second set of measurements, $AB = 100$ ft., $\angle DAC = 25°$, and $\angle DBC = 39°$. Find the height of the tower as before. What is the difference between the two results?

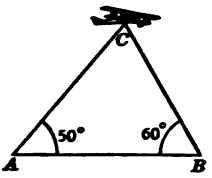


FIG. 40

6. Two observers at A and B in the same vertical plane with an airplane at C observe it at angles of 50° and 60°, respectively. The observers are 440 yards apart. Make a drawing to scale and find the height of the airplane.

7. A hostile ship at S is observed from two forts A and B $2\frac{1}{4}$ miles apart. The $\angle A$ is 75°, and $\angle B$ is 60°. The guns of each fort have a range of 3 miles. Is the ship within range of either fort?

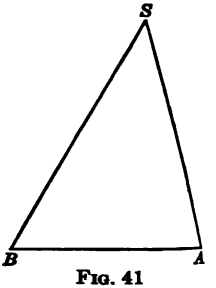


FIG. 41

8. A ladder 20 ft. long is set against a building. When the ladder is safe, the angle between the ladder and the building is not less than 15°. What is the highest point that can safely be reached by the top of the ladder?

CHAPTER V

VOLUMES AND SURFACES OF SOLIDS

2. Comparison of rectangular solids. This block of
d with square corners is 6 inches long, 2 inches wide, and
ches high.

. solid of this shape is called a
angular solid.

'he dimensions of the solid
ve are written 6″×2″×3″, and
is read six inches by two inches
hree inches.

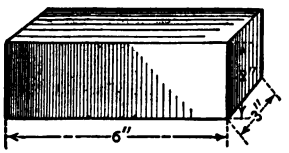


FIG. 42

nto how many blocks each 2″×2″×3″ can this block be
?

. box is 6″×4″×4″ (inside measurement). How many
ks each 3″×2″×2″ can be laid along one edge of the
tom if laid with the long edge of the block along the long
e of the box? How many such rows will it take to cover
bottom? How many such layers will it take to fill the
? How many blocks are required to fill the box?

nother box is 6″×4″×4″. How many blocks each
<2″×1″ can be laid along one edge of the bottom of this
? How many blocks are required for one layer on the
tom? How many layers to fill the box? How many
ks to fill the box?

. rectangular solid whose length, breadth, and thickness
all equal is called a **cube.**

. cube each of whose dimensions is one inch is called an
cube, or a **cubic inch.** A cube each of whose dimen-
s is 3 inches is called a three-inch cube, and so on.

Draw a picture of a nine-inch cube, using an eighth of an inch for an inch in measuring the nearest edge.

How many three-inch cubes can be laid along one edge of the bottom of a hollow nine-inch cube? How many three-inch cubes does it take to cover the bottom? How many to fill the nine-inch cube? In the same way find how many four-inch cubes in an eight-inch cube.

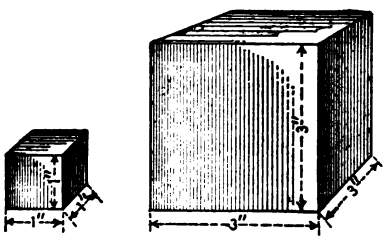


FIG. 43

A rectangular solid is usually measured by computing the number of cubes of a certain size it contains. If the dimensions of the solid are measured in inches, the cubic inch is used as the unit of measure.

By the volume of a solid is meant the number of cubes it contains whose edge is the unit of length with which the dimensions of the solid are measured. Thus, the volume of a solid $2' \times 3' \times 5'$ means the number of cubic feet it contains.

43. Board measure. In measuring lumber it is not convenient to use a cube as the unit of measure. Instead, a solid *1 foot long, 1 foot wide* and *1 inch thick* is used. This unit is called a **board foot.**

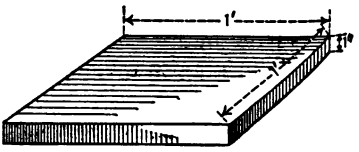


FIG. 44

How many board feet in a board 1 inch thick, 1 foot wide, and 2 feet long? In a board 1 foot wide, 1 inch thick, and 5 feet long? In a board 2 feet wide, 1 inch thick, and 3 feet long?

How many board feet in a board 1 inch thick, $\frac{1}{2}$ of a foot wide, and 2 feet long? One inch thick, 6 inches wide, and 6 feet long?

It is usual to speak of a number of feet of lumber when a number of board feet is meant.

How many feet of lumber in a piece 2 inches thick, 1 foot wide, and 10 feet long?

Make a rule for computing the number of feet in a piece of lumber. This rule should mention the units in which the length, width, and thickness are measured.

Make a formula from this rule, calling the length l, the width w, and the thickness t, and assuming that l and w are numbers of feet while t is a number of inches.

Rule. *To find the number of board feet in a piece of lumber multiply together the number of feet in the length, the number of feet in the width, and the number of inches in the thickness.*

The length of a board is usually given in feet, while both the width and thickness are given in inches. To reduce the width to feet it is only necessary to divide by 12, hence the following

Rule. *To find the number of board feet in a board multiply together its length in feet, its width in inches, and its thickness in in hes and divide by 12.*

These processes should first be indicated, then cancellation used.

EXAMPLE. How many board feet in a board $15' \times 8'' \times 2''$?

SOLUTION. $$\frac{\overset{5}{\cancel{15}} \times \overset{2}{\cancel{8}} \times 2}{\cancel{12}_{\cancel{4}}} = 20.$$

There are 20 board feet in the board.

In measuring lumber in board feet boards less than one inch thick are counted one inch thick.

Thus, a board 4″ wide, 12′ long, and $\frac{3}{4}$″ thick is counted as $\frac{4 \times 12 \times 1}{12}$ board feet, or 4 board feet.

Exercise 45

1. A brick is $8'' \times 4'' \times 2''$. How many bricks will it take to fill a box $2' \times 1' \times 1'$?

2. How many bricks can be loaded in a wagon bed $8' \times 2'\ 8'' \times 10''$?

3. Making no allowance for mortar, how many bricks are required for a foundation $22' \times 3' \times 16''$?

4. How many board feet in a board 3′ long, 1′ wide, and 2″ thick?

5. How many feet in a piece of lumber $3' \times 6'' \times 2''$?

6. How many feet in a two-by-four 8′ long? (A two-by-four is 2″ thick and 4″ wide.)

7. How many feet in a four-by-four 16′ long?

8. How many feet of one-half-inch flooring are required for a floor $18' \times 24'$, making no allowance for waste?

9. The lumber bill for a certain house calls for 78 pieces $2'' \times 4'' \times 9'$ and 24 pieces $2'' \times 8'' \times 12'$. How much does this lumber cost at $4\frac{1}{2}$ cents a foot?

10. The backstop for a baseball diamond is 10′ high and 16′ long. It is built of six-inch boards, 1″ thick and 16′ long. Three posts are used, each $4'' \times 4'' \times 16'$. How many feet of lumber are required?

11. How many feet in a wagon load of lumber 12′ long 3′ 4″ wide, and 2′ 6″ high if the boards are a half-inch thick?

12. How many feet of lumber are required to make a covered box whose outside dimensions are $16'' \times 26'' \times 45''$, using 1-inch lumber and allowing $\frac{3}{4}$ of a foot waste in squaring? How much will this lumber cost at \$65 a thousand?

13. Lumbermen use the formula, $V = \frac{(B+b)l}{2}$, for finding the number of cubic feet in a log, where B and b are the areas of the ends of the log in square feet, and l is its length in feet. Find the number of cubic feet in a log whose ends are 16 in. and 18 in. in diameter and whose length is 12 ft.

14. Use the formula given in the previous problem and find the number of cubic feet in each of the following logs:

	Diameter of Small End	Diameter of Large End	Length
(*a*)	20″	24″	12′
(*b*)	26″	32″	18′
(*c*)	30″	37″	20′

15. The Doyle Rule for finding the number of board feet in a log is the one most widely used in the United States. If N is the number of board feet in a log, d the diameter of the small end in inches, and l the length in feet, then by the Doyle Rule, $N=\left(\frac{d-4}{4}\right)^2 l$. Find by this rule the number of board feet in a log if the diameter of the small end is 18 in. and the length 16 ft.

16. Use the Doyle Rule and find the board feet in logs with the following dimensions:

	Diameter of Small End	Length
(*a*)	28″	20′
(*b*)	25″	30′
(*c*)	30″	18′

44. Volumes of rectangular solids. With inch cubes make a solid 5″ long, 4″ wide, and 3″ high. How many cubic inches in the row along AB, Figure 45? How many such rows in the bottom layer? How many cubic inches in this layer? How many layers? How many cubic inches in the solid?

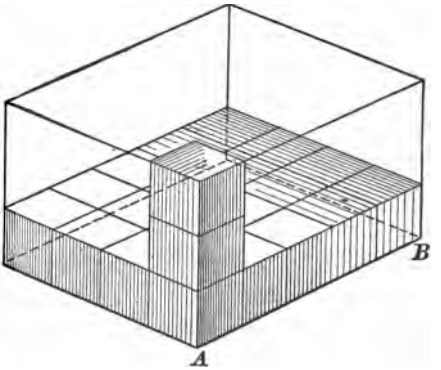


Fig. 45

From the answers to these questions we see that the volume of the solid is

$$3 \times 4 \times 5 \text{ cu. in.} = 60 \text{ cu. in.}$$

Answer similar questions concerning a solid $6\frac{1}{2}''$ long, $5''$ wide, and $4''$ high; also about a solid $6\frac{1}{2}'' \times 4\frac{1}{2}'' \times 6''$; also about a solid $8\frac{1}{2}'' \times 5\frac{1}{3}'' \times 9\frac{1}{4}''$.

Exercise 46

1. The volume of the solid $3'' \times 4'' \times 5''$ may be found by taking the product $5 \times 4 \times 3$ cu. in. Point out a part of the solid in Figure 45 whose volume is 3 cu. in. Point out a part whose volume is 4×3 cu. in.

2. The volume of this solid may also be found by taking the product $3 \times 4 \times 5$ cu. in. Point out a part of the solid whose volume is 5 cu. in. Point out a part whose volume is 4×5 cu. in.

Find the volumes of rectangular solids with the following dimensions:

3. $8''$, $12''$, $15''$.
4. $4''$, $9'$, $30'$.
5. $7\frac{1}{2}'$, $18'$, $35'$.
6. $3'\ 6''$, $8'\ 4''$, $20'$.
7. $10\frac{1}{2}''$, $4\frac{1}{4}''$, $12''$.
8. 3 yd., 6 yd. 2 ft., 5 yd.
9. $4.3''$, $9.6''$, $11.1''$, correct to .01 cu. in.
10. $25.1'$, $14.3'$, $75.8'$, correct to .01 cu. ft.
11. $8.3'$, $5.25'$, $7.4'$, correct to .01 cu. ft.
12. $22'$, $22'$, $4'\ 8''$, correct to .1 cu. yd.
13. A lot 100 ft. by 60 ft. is to be filled in with earth to a depth of 1 ft. How many loads will be required if a load is 1 cu. yd.?
14. A cellar is 20 ft. long, 18 ft. wide, and 8 ft. deep. How many cubic yards of earth must be removed? What is the cost of excavating at 50¢ a cubic yard?
15. A flat car is 40 ft. 4 in. long, 8 ft. 6 in. wide, and 3 ft. 6 in. deep. How many cubic yards of gravel will it hold if it is level full?

16. The inside dimensions of a concrete watering trough are 8′ 6″ by 2′ 6″ by 1′ 6″. How many gallons will it hold? Allow $7\frac{1}{2}$ gallons to 1 cu. ft.

17. The outside dimensions of the trough of the preceding problem are $9' \times 3' \times 2'$. How many cubic feet of concrete are needed to make it?

18. An ice house is 30 ft. long, 18 ft. wide, and 15 ft. high. How many cubic feet of ice will it hold, allowing $1\frac{1}{2}$ ft. on the sides and ends and 2 ft. above and 2 ft. below for sawdust?

19. A grain car is 30 ft. long and 8 ft. 6 in. wide. It is filled to a depth of 5 ft. with wheat. How many bushels of wheat does it then contain? Find the answer both by using 2150.4 cu. in. for one bushel, and by using $\frac{5}{4}$ cu. ft. for one bushel. What is the difference between the two answers?

20. This car is filled through a chute the end of which is 8 in. square. The wheat runs through the chute at the rate of 10 ft. a second. How long will be required to fill the car to the depth of 5 ft.?

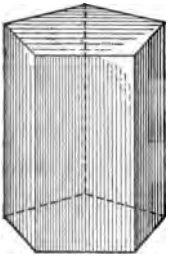
21. There are 30 pupils in a room 32 ft. long, 24 ft. wide, and 14 ft. high. How many cubic feet of air in the room for each pupil?

22. Fresh air is sent into this room through a ventilator shaft that is $1' \times 1'$ 6″. The air moves through the ventilator shaft at the rate of 10 ft. a second. How many cubic feet of air come into the room in one second? How many cubic feet of air come into the room for each pupil per minute?

23. The surface of a pond is $2\frac{1}{2}$ acres. Ice is frozen over it to a depth of 8 in. A cubic foot of water weighs $62\frac{1}{2}$ lb., and the specific gravity of the ice is .92. How many tons of ice on the pond? Answer to the nearest ton.

24. The average annual rainfall in Central Illinois is 38 in. How many gallons is that per square foot?

45. Volumes of prisms and cylinders. The surfaces that bound a solid are called its **faces.**

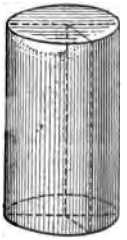


Prism
FIG. 46

A **prism** is a solid two of whose faces are equal and parallel polygons and whose other faces are parallelograms.

The equal and parallel faces are called the **bases** of the prism.

The other faces of the prism are called the **lateral surfaces.** Point out prisms in the schoolroom.



Cylinder
FIG. 47

Figure 47 is an example of a **cylinder.** The bases are equal and parallel circles. The **altitude** of a prism or a cylinder is the perpendicular distance between the bases. Point out cylinders in the schoolroom.

If the lateral surface of a prism or a cylinder is perpendicular to the bases, the prism or cylinder is called a **right prism** or a **right cylinder.**

Exercise 47

1. If the base of this prism, Figure 48, contains 12 sq. in., what is the volume of a portion of it that is 1 in. high? What is the volume of the prism if it is 10 in. high? How can you find the volume of a right prism if you know the area of the base and the altitude?

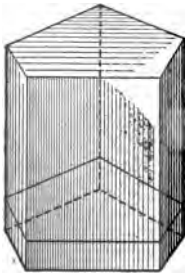


FIG. 48

2. The radius of the base of this cylinder is 4 in. What is the area of the base? What is the volume of a portion of this cylinder that is 1 in. high? What is the volume of the cylinder if the altitude is 6 in.? What is the formula for finding

the area of a circle when the radius is given? Make a rule for finding the volume of a cylinder when the radius and altitude are given.

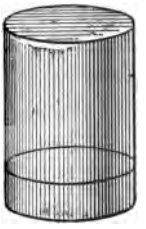


FIG. 49

3. Point out examples of prisms in the schoolroom and tell how many faces each has.

4. Point out solids that are not prisms or cylinders, and state why they are not.

5. Do you see any solids that are bounded partly by plane surfaces and partly by curved surfaces?

Rule for the volume of a prism. *The volume of a prism equals the product of the base by the altitude.*

Formula. $V = bh$.

Rule for the volume of a cylinder. *The volume of a cylinder equals π times the square of the radius times the altitude.*

Formula. $V = \pi r^2 h$.

Exercise 48

Find the volumes of cylinders with the following radii and altitudes:

1. Radius 3″, altitude 10″, correct to .1 cu. in.
2. Radius 2′, altitude 5′, correct to .01 cu. ft.
3. Radius 8′, altitude 16′, correct to .01 cu. ft.
4. Radius 3.3″, altitude 12″, correct to .1 cu. in.

Find the volume of each of the following prisms:

5. Base 40 sq. in., altitude 36 in.
6. Base 76.5 sq. ft., altitude 6 ft. 4 in.
7. Base 12 ft. square, altitude $9\frac{1}{2}$ ft.
8. The base is a rectangle 8 in. by 9 in., and the altitude is 2 ft. 3 in. Find the volume.
9. Find the altitude of a prism whose volume is 360 cu. ft. and whose base is 80 sq. ft.

10. Find the area of the base of a prism whose volume is 96 cu. in. and whose altitude is 15 in.

11. The radius of a cylinder is 2 in. and the volume 36 cu. in. Find the altitude.

12. A cylindrical pail is 7 in. in diameter and 6 in. deep. Does it contain a gallon?

13. The standard bushel is $18\frac{1}{2}$ in. in diameter and 8 in. deep. How many cubic inches does it contain?

14. The inside diameter of a water pipe is 2 in. How many gallons will flow through this pipe in one hour if the water flows at the rate of 30 ft. a minute?

15. A steel wire is $\frac{3}{16}$ of an inch in diameter. Find the weight of a mile of this wire, its specific gravity being 7.83.

16. A cylindrical cistern is 9 ft. in diameter and 10 ft. deep. How many barrels of water will it hold? (A barrel is $31\frac{1}{2}$ gallons. Assume $7\frac{1}{2}$ gal. = 1 cu. ft.)

17. A bushel of corn in the ear is approximately $2\frac{1}{4}$ cu. ft. How many bushels of corn in the ear can be put into a bin 24 ft. long, 7 ft. 6 in. wide, and 9 ft. deep?

18. A wagon bed is 10 ft. long, 3 ft. wide, and 26 in. deep, inside measurement. How many bushels of corn in the ear will it hold? How many bushels of wheat?

19. How many cubic yards of earth must be removed in digging a trench 4 ft. wide at the top, 2 ft. wide at the bottom, 5 ft. deep, and 10 rods long?

20. What is the weight of a column of water 3 in. in diameter and 12 ft. high?

21. In pumping water from a well there must be lifted at each stroke a column of water reaching from the level of the water in the well to the pump spout. What is the weight of the column of water to be lifted in pumping water if the diameter of the column is 3 in. and if the column is 20 ft. deep?

22. A vessel 12 in. in diameter is partly filled with water. A piece of stone of irregular shape is submersed in the water, raising the level $3\frac{1}{2}$ in. Find the volume of the piece of stone.

23. The end of a steel Z-bar has the dimensions and form given in Figure 50. Find the area of this end. Find the volume of such a bar 1 ft. long.

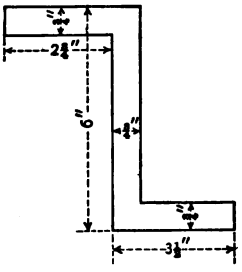


FIG. 50

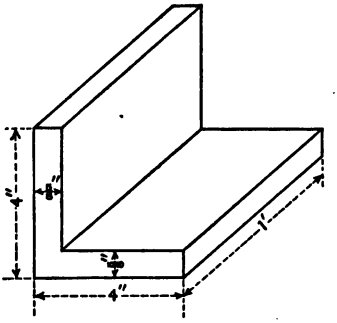


FIG. 51

24. A steel angle has the form and dimensions given in Figure 51. Find its volume.

25. Find the volume of a piece of lead pipe 10 ft. long, 1 inch thick, and an inside diameter of 2 in. The specific gravity of the lead is 11.3. Find the weight of this pipe.

26. In making estimates, 35 cu. ft. of hard coal, or 45 cu. ft. of soft coal are called a ton. How many tons of soft coal can be put into a bin 10 ft. square and 8 ft. deep filled to within 2 ft. of the top? How many tons of hard coal?

27. Find the volume of an iron plate of the form of Figure 52 if the plate is $\frac{5}{8}$ in. thick.

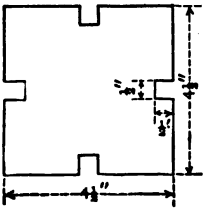


FIG. 52

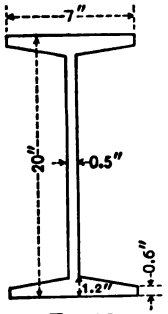


FIG. 53

28. Find the volume of a steel beam 24 ft. long, a cross section of which is given in Figure 53. Find the weight of this beam if the specific gravity is 7.83.

29. Find the volume of the iron ring of Figure 54, if the outer radius is 8 in., the inner radius 4 in., and the thickness $\frac{3}{8}$ in.

30. Find the volume of this plate, Figure 55, if the diameter of the plate is 10 in., the diameter of the holes 1 in., and the thickness $\frac{1}{2}$ in.

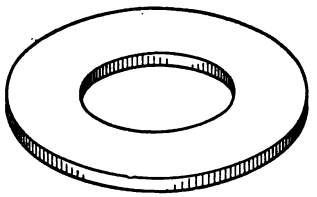


FIG. 54

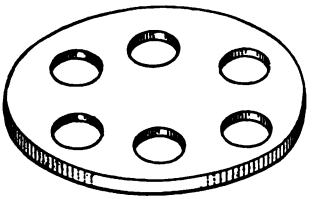


FIG. 55

46. Area of the surfaces of prisms and cylinders. In many problems of painting and building it is necessary to be able to find the area of the surfaces of cylinders and prisms.

Exercise 49

1. How many faces has a rectangular solid? What is the form of these faces? What form have the faces of a cube?

2. What is the total surface of a cube the edge of which is 6 in.?

3. What is the total surface of a brick which is 2 in. by 4 in. by 8 in.?

4. Compare the surfaces and the volumes of a 4-inch cube, and a rectangular solid 2 in. by 4 in. by 8 in. What is the ratio of the volumes? Of the surfaces?

5. What is the total surface of a rectangular solid of length l, width w, and thickness t? Write a formula where S represents the surface. Solve this formula for l, supposing S to be known.

6. Write a formula for the surface of a cube whose edge is e.

7. How many edges has a rectangular solid? What is the sum of the edges of the brick in exercise 3? Of the cube in exercise 6? Of the solid in exercise 5?

8. Cut a form like this pattern, Figure 56, from stiff paper or light cardboard. Fold along the dotted lines, paste together, and thus make a rectangular solid. What will be its dimensions?

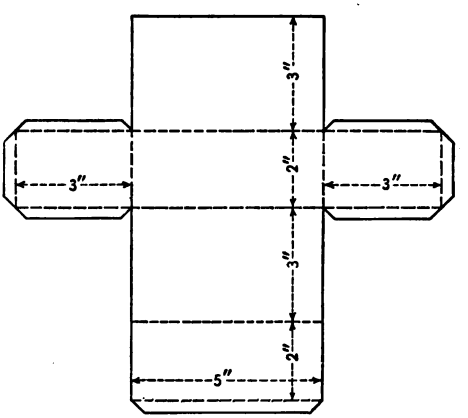


FIG. 56

9. In a similar way make a three-inch cube.

10. A rectangular piece of paper 12″ by 8″ is rolled into a cylinder by putting the two longer edges together. What are the circumference and the altitude of this cylinder?

11. A tin can 4 in. in diameter and 6 in. high is covered with paper. The paper is slit along the seam of the can, unrolled and spread out on a plane surface. What are the dimensions and area of the rectangle thus formed?

12. The circumference of a cylinder is c and its altitude is h. If the curved surface is rolled out into a rectangle, what are the base and altitude of the rectangle? If L is the lateral surface of the cylinder, show that $L=ch$. If r is the radius of the cylinder, show that $L=2\pi rh$. Solve this formula for r; for h.

13. Find the lateral surface of a cylinder 6 in. in diameter and 8 in. long.

14. How much sheet iron is required to make a piece of stove pipe 6 in. in diameter and 4 ft. long, allowing $\frac{1}{2}$ in. for the seam?

15. Cut a form like this pattern, Figure 57, from stiff paper or cardboard. Paste the edges together and thus form a cylinder. How long is the circumference? Answer correct to the nearest $\frac{1}{16}$ in.

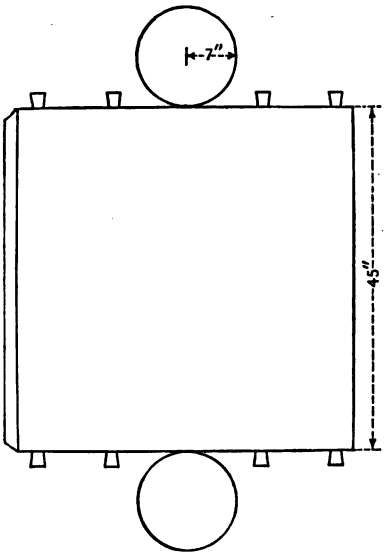


FIG. 57

16. Find the total surface of the above cylinder, that is, the area of the lateral surface plus the area of the two bases.

17. The radius of a cylinder is 4 in. and its altitude is 10 in. Let L represent the lateral surface and S represent the total surface. Show that $L=2\times\pi\times4\times10$ sq. in., and $S=2\times\pi\times4^2$ sq. in. $+2\times\pi\times4\times10$ sq. in.

18. If r is the radius, a the altitude, L the lateral surface, and S the total surface of a cylinder, show that $L=2\pi rh$ and that $S=2\pi rh+2\pi r^2$. Solve the last formula for h.

19. What is the total surface of the tank of a sprinkling wagon which is $3\frac{1}{2}$ ft. in diameter and 7 ft. long?

20. How much paint will be required for the lateral surface of a cylindrical silo which is 16 ft. in diameter and 30 ft. high, allowing 1 gallon to 250 sq. ft.?

21. What is the grinding surface of a grindstone that is 2 ft. in diameter and $3\frac{1}{2}$ in. thick?

22. A room is heated by 420 ft. of pipe whose outer diameter is 3 in. What is the total heating surface?

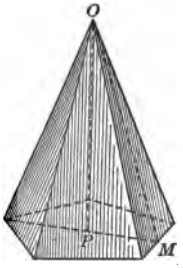
47. Volume and surface of a pyramid. A **pyramid** is a solid whose base is any polygon and whose lateral surfaces are triangles with a common vertex, which is called the **vertex** of the pyramid.

The **altitude** of a pyramid is the perpendicular from the vertex to the base.

If the base of a pyramid is a regular polygon and if the edges meeting at the vertex are all equal, the pyramid is called a **regular pyramid.**

The perpendicular from the vertex to one edge of the base of a regular pyramid is called the **slant height** of the pyramid.

In Figure 58, O is the vertex, OP the altitude, and OM the slant height of the regular pyramid.



Pyramid
FIG. 58

Take a hollow prism and a hollow pyramid which have equal bases and equal altitudes. It may be found by trial that the prism will hold three times as much sand or other substance as the pyramid. Since the volume of the prism is the product of its base and altitude, we have this

Rule. *The volume of a pyramid is one third of the product of its base and altitude.*

If V is the volume, h the altitude, and b the area of the base, we have the formula $V = \frac{bh}{3}$.

Cut a form like Figure 59 from light cardboard or stiff paper. Fold along the dotted lines, paste together the edges, and thus form a pyramid. What kind of pyramid does this give? The lateral faces of a regular pyramid are what kind of triangles? Are all the faces of the same size? In Figure 59 what is the area of one lateral face? What is the total lateral area? What is the perimeter of the base? What must the perimeter of the base be multiplied by to give the lateral area? If A is the lateral area of a regular pyramid, s the slant height, and p the perimeter of the base, then $A = \frac{sp}{2}$.

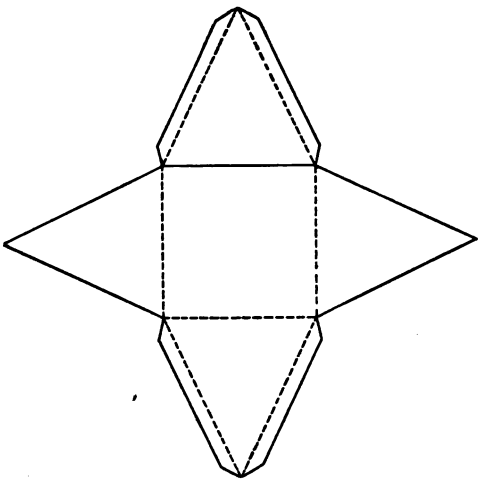


FIG. 59

Exercise 50

1. Find the volume of a pyramid whose base is 185 sq. in. and whose altitude is 18 in.

2. Find the volume of a pyramid whose base is a square with an edge of 15 in. and whose altitude is 8 in.

3. Is the pyramid in the last exercise necessarily a regular pyramid?

4. Do you know of any examples of pyramids in the roofs of houses or in the tops of towers?

5. A brass paper weight has the form of a pyramid. Its base is 1 inch square and its altitude is $1\frac{3}{8}$ in. How many pounds of brass are required to make 100 such paper weights? The specific gravity of brass is 8.4.

6. The roof of a building 24 ft. square is in the form of the surface of a regular pyramid, whose slant height is 14.5 ft. The roof is covered with a composition roofing. How many square yards of this roofing are required if there is a waste of 5% of the area of the roof?

7. What is the difference in the number of square yards of roofing required for the two forms of roofs shown in Figure 60?

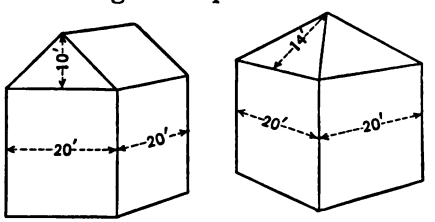


FIG. 60

8. The pyramid of Cheops in Egypt has a base 764 ft. square and an altitude of 480.75 ft. Find its volume.

9. How many square yards of canvas are required to make a tent in the form of a pyramid whose base is 10 ft. square and whose height is 9 ft.?

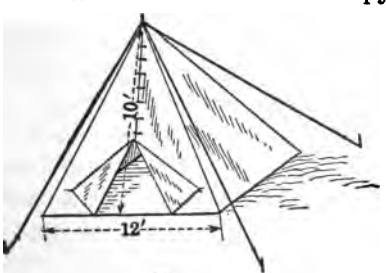


FIG. 61

10. How many square yards of canvas are required to make a square tent of the dimensions given in Figure 61?

11. Solve the formula $V = \frac{1}{3}bh$ for h.

12. Find the altitude of a pyramid whose volume is 84 cu. in. and whose base is 6 in. square.

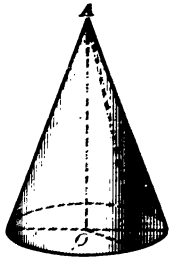
13. The base of a pyramid is an equilateral triangle whose side is 10 in. The volume of the pyramid is 5196 cu. in. Find the altitude.

14. Solve the formula $V = \frac{1}{3}bh$ for b.

15. Find the area of the base of a pyramid whose altitude is 14 ft. and volume 264 cu. ft.

48. Volume and surface of a cone. If a right triangle is rotated about the altitude as an axis, the figure traced is called a **cone.**

The kind of cone thus obtained is a **right circular** cone. It is the only kind of cone that we shall consider in this book. When we speak of a cone, we shall mean a right circular cone.



Cone
FIG. 62

The point A is the **vertex** of the cone.

The circle with the center O is called the **base** of the cone, and the other surface of the cone is called the **lateral surface.**

The perpendicular distance from the vertex to the base is the **altitude** of the cone.

The distance from the vertex to the edge of the base is the **slant height** of the cone.

Take a hollow cylinder and a hollow cone with equal bases and equal altitudes. It may be found by trial that the cylinder will hold three times as much sand or other substance as the cone. If the cylinder has the base b and the altitude a, what is its volume? What is the volume of the cone with the same base and the same altitude? Make a rule for finding the volume of a cone when the area of the base and the altitude of the cone are known. If the radius of the base is r, what is the area of the base?

The formula for the volume of a cone is

$$V = \tfrac{1}{3}\pi r^2 h,$$

where r is the radius of the base and h the altitude of the cone.

Cut a form like Figure 63 from stiff paper or light cardboard. If the edges OA and OB are then pasted together and the arc AB is pasted to the circle C, a cone is formed. The circle C then becomes the base of the cone.

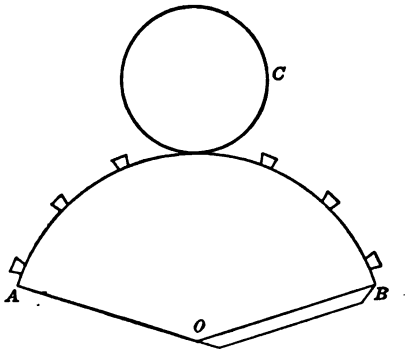


FIG. 63

This illustrates that if the lateral surface of a cone is slit along a straight line from the vertex to the base and then rolled out flat a sector of a circle is formed.

The area of a sector may be found by the same method as was used for finding the area of a circle. Cut the sector AOB (Figure 63) into smaller sectors and fit them together as in Figure 64. Let the pupil show from this illustration that *the area of a sector equals the radius times one-half the arc.*

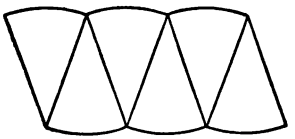


FIG. 64

In Figure 63 the arc AB is seen to be the same length as the circumference of the base of the cone, and the radius OA is the slant height of the cone. We have then the

Rule. *The lateral surface of a cone equals one-half the product of the circumference of the base by the slant height.*

If the radius of the base of a cone is r, the slant height s, and the lateral surface L, then

$$L = \pi r s.$$

If S is the total surface of the cone, then

$$S = \pi r s + \pi r^2.$$

Exercise 51

1. Find the volume of a cone, the radius of the base being 4 in. and the altitude 10 in.

2. Find the volume of an ice cream cone if the diameter of the base is $2\frac{1}{2}$ in. and the altitude 4 in.

3. How many square yards of canvas are required to make a conical tent 10 ft. in diameter and 9 ft. high, allowing 1 sq. yd. extra for seams and waste?

4. A conical roof has a diameter of 20 ft. and a slant height of 16 ft. Find its area.

5. Find the total surface of a cone if the radius of the base is 5 in. and the slant height is 22 in.

6. What is the lateral surface of an ice cream cone, the radius of the base being 1 in. and the slant height 4 in.?

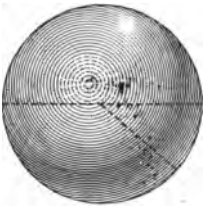
7. A cone and two pyramids have the same slant height, 24 in. The base of the cone is 8 in. in diameter ; the base of one pyramid is 8 in. square ; the base of the other pyramid is a regular hexagon 4 in. on a side. Find the lateral surface of the cone and of each of the pyramids. Which has the largest lateral surface? Which the smallest?

8. Find the altitudes of cones if the slant heights and the radii of the bases are the following numbers respectively: 5, 4 ; 10, 6 ; 13, 5 ; 20, 16 ; 17, 8 ; 2, 1.

9. How many cubic yards of sand in a pile, assumed to be a cone, 12 ft. in diameter and 4 ft. high?

10. A brass paper weight is in the form of a cone. The radius of the base is 1 in. and the altitude is 2 in. How many such paper weights can be made from a cubic foot of brass?

49. Volume and surface of a sphere. A **sphere** is a solid bounded by a curved surface every point of which is equally distant from a point within called the **center**.



Sphere

FIG. 65

Let a sphere be cut into hemispheres. Cover the curved surface and also the flat surface of a hemisphere with a cord, Figure 66. By comparing the length of the cord required to cover the curved surface with that required to cover the flat surface it will be seen that the curved surface of a hemisphere is twice the flat surface of the hemisphere. Since the area of the flat surface of the hemisphere is πr^2, we have

$$S = 4\,\pi r^2,$$

where S is the whole surface of the sphere.

Rule. *The area of a sphere equals four times π times the square of the radius.*

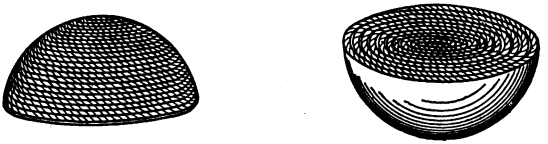


FIG. 66

It is proved in geometry that the formula for the volume of a sphere is $V = \frac{4}{3}\,\pi r^3$, where V is the volume, and r is the radius of the sphere.

Exercise 52

1. How many square inches in the surface of a globe that is 6 in. in diameter?

2. How many pounds of steel are required to make 1000 steel balls $\frac{1}{2}$ in. in diameter? The specific gravity of steel is 7.83.

3. Find the volume of the earth, assuming it to be a sphere 8000 mi. in diameter.

4. Find the volume of the largest sphere that can be cut from a 10-inch cube.

5. Find the volume of the largest cylinder that can be cut from a 10-inch cube.

Exercise 53. Problems about concrete work

In the following problems 1 sack of cement is to be counted as 1 cu. ft. A 1 : 2 : 3 mixture of cement, sand, and gravel means a mixture of 1 part of cement, 2 parts of sand, and 3 parts of gravel. Use the following prices : Cement, \$.40 a sack ; sand and gravel, \$1.10 a cubic yard ; cinders, \$.25 a cubic yard. It will be assumed that if a fraction of a sack of cement or a fraction of a cubic yard of the other materials is needed, a whole sack, or a whole cubic yard, will be bought.

1. Find the cost of the materials to make a concrete watering trough 8 ft. long, 4 ft. wide, and 3 ft. deep, inside measurements, with sides and bottom 6 in. thick, made of a 1 : 2 : 3 mixture of cement, sand, and gravel.

2. Find the cost of the materials for one mile of concrete sidewalk, 4 ft. wide, which has a base of cinders 4 in. thick, then a layer of a 1 : 7 mixture of cement and gravel 3 in. thick, and a top $\frac{3}{4}$ in. thick of a 1 : 2 mixture of cement and sand.

3. Find the cost of the materials for 1000 fence posts, 8 in. by 4 in. at the bottom, 4 in. square at the top, and 7 ft. long. They are made of a 1 : 3 mixture of cement and gravel, and each post is reënforced by 4 wires running through it lengthwise, each wire being 4 in. longer than the post. This wire costs 2.5¢ a pound. Twenty feet weigh a pound.

4. Find the cost of the materials for making 10 concrete gate posts 12 in. square at the bottom, 12 in. by 8 in. at the top, and 8 ft. long. They are made of a 1 : 3 mixture of

cement and gravel and each is reënforced by 4 steel rods costing 4¢ a foot.

5. What is the cost of the materials for the posts needed to fence a lot 132 ft. by 206 ft., placing the posts a rod apart, and having a gate 8 ft. wide in each of the long sides of the lot? The posts used are of the sizes given in problems 3 and 4.

6. An excavation 57 ft. 9 in. wide and 86 ft. 3 in. long is made for a building. The excavation is 5 ft. deep except for a trench 2 ft. wide running around the outside of the excavation, which is 8 in. deeper. Find the cost of making the excavation at 40 cents a cubic yard.

7. The trench, in the previous problem, is for the base, or footing, of the foundation of the building. This footing is 2 ft. wide and 8 in. deep, and is made of a 1 : 6 mixture of cement and gravel. Find the cost of the materials for the footing.

8. The foundation for this building is 84 ft. 7 in. long and 56 ft. 1 in. wide, outside measurements, and is 10 in. thick and 8 ft. 9 in. high. It is made of a 1 : 3 : 5 mixture of cement, sand, and gravel. Find the cost of the materials.

9. The basement of this building has a concrete floor which cost 10½ cents a square foot. Find its cost.

10. A circular grass plot 120 ft. in diameter is surrounded by a concrete walk 6 ft. wide. This walk is constructed in the same way as the one in Problem 2. Find the cost of the materials.

11. What is the cost per square foot of the materials for the walk in the previous problem? Find the answer correct to .01 cent.

12. If the circular grass plot of problem 10 had twice as great a diameter, what would be the cost of the materials for the walk? How many times as much as for the plot whose diameter is 120 ft.?

Exercise 54. Problems about silos

Silos are used to store food for livestock. The contents of a silo are called **silage.** Corn is much used for silage. The corn is cut green, run through a cutter, and run into the silo. The silage is tramped down as tightly as possible so as to shut out the air. Silos are usually cylindrical in shape.

FIG. 67

If silage is exposed too long to the air, it becomes unfit for feed. Hence in building a silo care is taken to make it the proper size so that silage will be removed fast enough to prevent decay. The surface of the silage that is exposed to the air, which is the same as the area of one end of the silo, is called the " feeding surface."

1. A cylindrical silo is 18 ft. in diameter, inside measure, and 30 ft. high. Find its capacity in cubic feet.

2. If a cubic foot of corn silage weighs 40 lb., how many tons of corn silage will the silo in the previous problem contain?

3. How many cubic feet of corn silage in 100 tons?

4. How long will 100 tons of silage last 40 cows if each cow eats 25 pounds a day?

5. It is estimated that there should be about 6 square feet of feeding surface for each cow fed. How many square feet of feeding surface are needed for 25 cows? What is the diameter of a cylindrical silo that has this amount of feeding surface, correct to the nearest foot?

6. Silos are usually made with inside diameters of 12, 14, 16, 18, 20, or 22 ft. Which is the smallest of these sizes that

will give the amount of feeding surface needed in the preceding problem?

7. How many tons of silage are needed to feed 20 cows 200 days, feeding each cow 25 lb. daily? How many cubic feet of silage is this, counting 40 lb. to the cubic foot?

8. What must be the height of a silo with inside diameter 12 ft. to hold the amount of silage found in the preceding problem?

9. A cylindrical silo is made of concrete with walls 6 in. thick. The inside diameter is 14 ft. and the height is 30 ft. How many cubic feet of concrete are required?

10. Show that a cylindrical silo 18 ft. in diameter holds approximately as much as a square silo of the same height and 16 ft. square, inside measurement in each case. Find the difference in the amount of concrete to build one of each form, 30 ft. high, with walls 6 in. thick.

11. A $1 : 2\frac{1}{2} : 4$ mixture of cement, sand, and gravel is used to make the walls of silos. Find the cost of the materials for making the walls of each of the silos in problems 9 and 10 if cement costs 50¢ a cubic foot, sand \$1.10 a cubic yard, and gravel \$1.10 a cubic yard.

12. A wooden silo is 15 ft. in diameter, outside measurement, and 30 ft. high. Find the cost of the paint, at \$1.70 a gallon, for painting the outside, allowing 1 gallon to 250 sq. ft.

13. The silo in problem 12 has a conical roof whose slant height is 11 ft. and the radius of whose base is 9 ft. How many bundles of shingles are required for this roof, allowing 1000 shingles to 100 sq. ft.? A bundle contains 250 shingles.

CHAPTER VI

NEGATIVE NUMBERS

50. Meaning of negative numbers. In playing beanbags a player is allowed so many points for each of the circles into which he throws, but he loses a certain number of points for each throw that misses the board. His first throw counts 8, his second counts 5, his third throw misses the board, losing 10, his fourth counts 5, his fifth counts 6, and his sixth loses 10. In keeping the score the counts for him are marked $+$, meaning that these numbers are to be added to his score, while the counts against him are marked $-$, meaning that these numbers are to be subtracted from his score. His counts, then, are $+8$, $+5$, -10, $+5$, $+6$, -10. Find his final score.

The number $+8$ is read " plus 8 " or " positive 8." The number -10 is read " minus 10 " or " negative 10." What score does the next player make in two throws if his first count is $+7$ and the next $+5$? In three throws counting $+7$, $+5$, and -10? What score does a player make in four throws counting $+8$, $+6$, -10, $+2$? In two throws counting $+8$ and -10? In three throws counting $+7$, -10, $+6$?

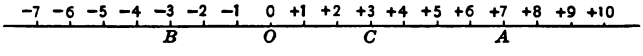


FIG. 68

It is easy to measure such scores along a straight line as in Figure 68. Such a line as this is called a **number scale.**

The counts $+7$, -10, and $+6$ may be found by counting to the right for the plus counts and to the left for the minus counts.

ırting at O, OA is $+7$, AB is -10, and BC is $+6$. The after any throw is the distance from the starting point the stopping point after that throw. After the first r his score is OA, or $+7$; after the second it is OB, or after the third it is OC, or $+3$.

poor player misses his first throw, ting 10 against him. What is his then? His second throw goes into ing counting 5, his third into the ring ting 2, his fourth misses, his fifth s 4, his sixth counts 1, and his ıth misses. What is his score then? ond player misses his first throw, his d goes into the ring counting 5, his into the ring counting 2, his fourth s, his fifth counts 4, his sixth counts ıd his seventh misses. What is his score? Show on a number scale his after each throw.

e thermometer scale is just such a line keeping temperature scores. As the increases the mercury rises and as the decreases the mercury falls. The temure score is the number of degrees e or below zero at which the mercury ls.

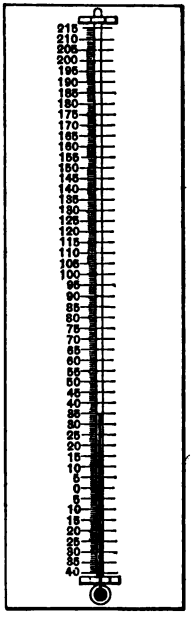


FIG. 69

Exercise 55

Combine these counts, made in playing a game, and each player's final score.

PLAYER A	PLAYER B	PLAYER C	PLAYER D
+20	+15	− 5	+15
+14	− 5	−10	+10
−25	− 8	+20	−14
−10	+25	+14	−11

2. On a number scale show the score of each player after each throw.

3. On a certain day the temperature started at $-8°$ and readings showed these changes: $-2°$, $-5°$, $+1°$, $+3°$, $+5°$, $+10°$, $+3°$, $+1°$, $-4°$. What was the final temperature?

On a number scale show the results of combining the following numbers :

4. $+4, +7, -3.$
5. $+8, -7, -4.$
6. $-3, -6, -2.$
7. $-6, +8, +2.$
8. $-7, +4.$
9. $-1, -1, +1, +2, -3.$
10. $+4, +7, -12, -8.$
11. $+8, -7, -5, +6.$
12. $-2, +7, -4, +3, -8.$
13. $-2, -1, 0, +1, +2.$
14. $-9, -11, +13, -2.$
15. $+5, -7, -1, -2, +16.$

16. In keeping account of business ventures it is customary to call the expenditures minus amounts and the receipts plus amounts. A boy goes into the chicken business with $+\$15$. His account shows the following amounts : $-\$2$; $-\$1.75$; $+\$.50$; $+\$1$; $+\$3$; $-\$.25$. How much has he then?

51. Adding signed numbers. Find the final score of a player whose counts are $+8$, -10, $+5$, $+2$, -6.

Combining $+8$, -10, $+5$, $+2$, and -6 in this way is called **adding** them, and the result is called their **sum.**

On the number scale these numbers may be added by beginning at zero and counting 8 spaces to the right, then 10 spaces to the left, then 5 spaces to the right, then 2 more spaces to the right, then 6 spaces to the left. The sum is then the number represented by the distance and the direction from the zero point to the stopping point.

Make a scale and on it find the sum of the following :

$+4$	$+2$	$+6$	-4	-2	-6	-2	$+4$	$+6$	$+5$	$+5$
$+7$	$+3$	$+8$	-7	-3	-8	$+3$	-7	-2	-3	-8

Can you make a rule for finding the sum of two numbers having the same sign? Two numbers having unlike signs?

The value of a number without regard to its sign is called its **absolute value.** Thus, the absolute value of $+4$, also of -4, is 4.

Rule. *To add two numbers having like signs add their absolute values and give their sum the common sign.*

To add two numbers having unlike signs find the difference of their absolute values and give this difference the sign of the number having the greater absolute value.

If the sign of a number is omitted, the plus sign is understood. Thus, $+8a$ is often written $8a$.

Exercise 56

Add:

1. $+7$, $+3$
2. $+7$, -3
3. -7, $+3$
4. -7, -3
5. -15, $+7$
6. -1, $+1$
7. $+6$, -6
8. $+4$, -11
9. -8, -3, -4
10. $+6$, -3, -4
11. -12, $+3$, -4
12. $+18$, -14, $+12$
13. -9, -6, -8
14. -4, $+10$, $+3$
15. -22, $+32$, -10
16. $-2a$, $+3a$
17. $+12n$, $-9n$
18. $-4a^2$, $-16a^2$
19. $+15ab$, $-6ab$
20. $+3d$, $-d$
21. $-7n$, $+n$
22. $-14mn$, $-6mn$, $8mn$
23. $-3a$, $7a$, $-6a$
24. $4r$, $-r$, $-r$
25. $6s^2$, $-12s^2$, $6s^2$
26. $-7rs$, $8rs$, $-7rs$
27. r, $-r$, $-2r$

52. Subtraction of signed numbers. By subtracting 7 from 12 we have always meant finding the number which added to 7 gives 12. This meaning is easily applied to signed numbers. Since $+7$ added to $+5$ gives $+12$, then $+7$ subtracted from $+12$ gives $+5$.

What number added to -3 gives -7?
What number added to -4 gives -2?
What number added to $+6$ gives $+2$?

In the following exercises subtract the lower number from the upper by finding the number which added to the subtrahend gives the minuend.

$+8$	-12	$+5$	-3	-8	$+3$	$+3$	-5	-8
$+3$	-4	$+7$	-5	-12	-2	-5	$+2$	$+10$

Check the preceding results by adding the subtrahend and remainder.

In each of the preceding examples think the sign of the subtrahend changed and add the resulting number to the minuend. Compare the result with that already found.

Can you make a rule for subtraction?

It is interesting to see how the rule applies in finding the difference between two readings on the thermometer scale.

EXAMPLE. Find the difference between a reading of $+29°$ and a reading of $+35°$.

SOLUTION. Starting at $+29°$ and going to $+35°$ we pass over $6°$ and go in the positive direction. Therefore the result of subtracting $+29°$ from $+35°$ is $+6°$. This result will also be obtained by adding $-29°$ and $+35°$ on the scale, that is, by going $+35°$, then $-29°$ and reading from 0 to the stopping point.

Find the remainder when -6 is subtracted from $+2$ by starting at -6 on the scale and counting the spaces to $+2$. Then find it by adding $+6$ to $+2$ on the scale. This illustrates the

Rule for subtraction. *Change the sign of the subtrahend and add.*

Exercise 57

Subtract :

1.	$+8$	6.	$+12$	11.	-1	16.	$+2b$	21.	-34
	-2		$+5$		-1		$-3b$		$+34$
2.	$+3$	7.	-14	12.	0	17.	$-6a^2$	22.	-7
	$+5$		-8		-6		$+5a^2$		-7
3.	$+6$	8.	-1	13.	-2	18.	$+c$	23.	$+a^2b$
	$+9$		$+1$		$+2$		$+2c$		$-a^2b$
4.	-9	9.	-19	14.	-18	19.	$-2mn$	24.	-3.5
	-7		-19		-7		$-7mn$		$+7.5$
5.	-4	10.	$+15$	15.	$+25$	20.	-624	25.	-0.1
	$+9$		$+8$		-25		-625		$-.1$

53. Applications of addition and subtraction of signed numbers.

We have seen that numbers above and below zero on the thermometer are represented by + and − numbers, respectively.

In the same way \$15 of debt may be represented by −\$15 and \$15 of credit by +\$15.

East longitude is usually called − and west longitude +.

Ten years B.C. may be called −10 years and 10 years A.D. may be called +10 years.

In many other cases opposite kinds of numbers may be indicated by the plus and the minus signs.

Exercise 58

1. A boy's account shows +\$25 and −\$15. What is his balance?

2. To how much does a debit of \$40 and a credit of \$25 amount?

3. A vessel is in 15° east longitude and sails 25° westward. What is then its longitude?

4. A vessel is in 12° south latitude and sails 20° northward. What is its latitude then?

5. A vessel is in latitude $-12°$ and sails $-15°$. What is its latitude then?

6. A man was born in the year 18 B.C. and lived 52 years. In what year did he die?

7. How old was a man at his death who was born 4 B.C. and died 29 A.D.?

8. A train runs south 15 mi. an hour and a brakeman walks forward on it at the rate of 3 mi. an hour. How fast and in what direction is the brakeman moving?

9. How fast is the brakeman of the preceding problem moving if he walks toward the rear of the train, the rates being the same as before?

10. A pupil takes three tests to determine his standing in a subject. In the first his mark is 8 points below passing, in the second it is 17 points above passing, and in the third it is 6 points below. Does he pass? By what margin?

11. In a football game the ball moves 12 yards toward the north goal, then 8 yards toward the south, then 5 yards north and stops. How far and in what direction is the stopping from the starting point?

12. In a certain election there are 38 votes for and 27 against a certain candidate. What is the candidate's plurality?

13. Julius Cæsar was born 100 B.C. and died 44 B.C. How old was he when he died?

14. Octavius Cæsar was born 63 B.C. and died 14 A.D. How old was he when he died?

15. A boy has debts of 10¢, 25¢, \$2.25, and 40¢; he has credits of 85¢, 60¢, \$1.25, and 50¢. Represent these debts and credits by signed numbers and find their sum.

16. A merchant has debts of \$100, \$500, and \$750; he has credits of \$50, \$600, \$10, and \$2000. Represent these debts and credits by signed numbers and find their sum.

Multiplication of signed numbers. The temperature
ach hour for 4 hours. What was the total fall?
emperature changed $-2°$ each hour for 5 hours.
as the total change? If it changed $+6°$ each hour
urs, what was the total change? If it changed $-7°$
ır for 6 hours?

upil has known that 3×7 means that 7 is to be added
beginning with zero. We may think $+3 \times +7$ to
e same as 3×7 and do to the $+7$ what the sign before
lls us to do, that is to add $+7$ three times. Thus
$'=0+(+7)+(+7)+(+7)=+21.$

$+3 \cdot -7=0+(-7)+(-7)+(-7)=-21.$

rly $-3 \cdot +7$ means that $+7$ is to be subtracted three
ginning with zero. Thus,

$$-3 \cdot +7=0-(+7)-(+7)-(+7)=-21.$$
$$-3 \cdot -7=0-(-7)-(-7)-(-7)=+21.$$

s
$$(+3)(+7)=+21.$$
$$(+3)(-7)=-21.$$
$$(-3)(+7)=-21.$$
$$(-3)(-7)=+21.$$

this the pupil should get the following

To find the product of two factors multiply their values. If the two factors have like signs the product f unlike signs the product is minus.

Exercise 59

$)(+3)=?$	**8.** $(-1)(+3)=?$	**15.** $(+8)(-7)=?$
$)(-5)=?$	**9.** $(+9)(-2)=?$	**16.** $(-9)(-6)=?$
$)(+5)=?$	**10.** $(-7)(-5)=?$	**17.** $(-a)(-b)=?$
$)(-4)=?$	**11.** $(-11)(+7)=?$	**18.** $(+a)(+b)=?$
$)(-1)=?$	**12.** $(-15)(-1)=?$	**19.** $(+a)(+a)=?$
$)(-9)=?$	**13.** $(+6)(+1)=?$	**20.** $(-a)(-a)=?$
$)(-2)=?$	**14.** $(-6)(-1)=?$	**21.** $(-2)(+a^2)=?$

55. Division of signed numbers. When given the product of two numbers and one of the numbers, the process by which the other number is found is called **division.** $12 \div 3 = 4$ because $3 \cdot 4 = 12$.

Supply the missing number:

(*a*) $(\quad)(+3) = +12$.
(*b*) $(-3)(\quad) = +12$.
(*c*) $(+4)(\quad) = -12$.
(*d*) $(-4)(\quad) = -12$.

From (*a*) we see that $(+12) \div (+3) = +4$.
From (*b*) we see that $(+12) \div (-3) = -4$.
From (*c*) we see that $(-12) \div (+4) = -3$.
From (*d*) we see that $(-12) \div (-4) = +3$.

As in multiplication, so also in division, *the quotient of two numbers with like signs is positive and with unlike signs is negative.* This statement is sometimes shortened to *like signs give plus and unlike signs give minus.*

Exercise 60

1. $\dfrac{+24}{+3}$.
2. $\dfrac{-24}{-3}$.
3. $\dfrac{+24}{-3}$.
4. $\dfrac{-24}{+3}$.
5. $\dfrac{3 \cdot 4}{3}$.
6. $\dfrac{6 \cdot 7}{6}$.
7. $\dfrac{6 \cdot 7}{2}$.
8. $\dfrac{-3 \cdot 5}{-3}$.
9. $\dfrac{-3 \cdot 5}{5}$.
10. $\dfrac{-6 \cdot (+7)}{-2}$.
11. $\dfrac{(-12)(+15)}{-2}$.
12. $\dfrac{(-12)(+15)}{-3}$.
13. $\dfrac{(-6)(-8)(+10)}{-2}$.
14. $\dfrac{(-15)(+18)}{(-5)(-2)}$.
15. $\dfrac{(-3)(-3)(-3)}{-3}$.
16. $\dfrac{(-3)(-3)(-3)(-3)(-3)}{(-3)(-3)}$.
17. $\dfrac{(-3)^3}{-3}$.
18. $\dfrac{(-3)^5}{(-3)^2}$.
19. $\dfrac{-5^4}{+5^2}$.
20. $\dfrac{+5^4}{-5^2}$.
21. $\dfrac{(-1)^5}{(-1)^2}$.
22. $\dfrac{-1^5}{-1^2}$.

56. Multiplication and division of polynomials. In sections 20 and 21 the pupil has seen that,

$$a(b+c+d)=ab+ac+ad,$$
$$(a+b)(c+d)=ac+ad+bc+bd,$$

and has learned the

Rule. *To multiply two polynomials together multiply each term of the multiplicand by each term of the multiplier and add these products.*

This can now be applied when some of the terms of the polynomials are negative.

EXAMPLE. Multiply $2x-3$ by $3x+4$.

SOLUTION.

$$\begin{array}{r} 2x-3 \\ 3x+4 \\ \hline 6x^2-9x \\ \quad +8x-12 \\ \hline 6x^2-\ x-12 \end{array}$$

In section 23 the pupil learned to divide a polynomial by a monomial by dividing each term of the polynomial by the monomial. He can now apply this rule to negative numbers.

EXAMPLE. Divide $-4s^2+5rs-3s$ by $-s$.

SOLUTION. $(-4s^2+5rs-3s)\div(-s)=+4s-5r+3$, found by dividing $-4s^2$ by $-s$, then dividing $+5rs$ by $-s$, then $-3s$ by $-s$.

Exercise 61

1. $-6(x-3)=?$
2. $a(2a-b)=?$
3. $-2x(3x-7)=?$
4. $(a-2)(a-3)=?$
5. $(r-1)(r+2)=?$
6. $(m+3)(2m-4)=?$
7. $(2r-1)(r+1)=?$
8. $(a^2-4)(a^2+5)=?$
9. $(m-n)(m+n)=?$
10. $(3x-7y)(3x+7y)=?$
11. $(x^2+4)(x^2-4)=?$
12. $(a-b)(a-b)=?$
13. $(3a-b)(3a-b)=?$
14. $(x-y)^2=?$
15. $(-xy+2y)\div(-y)=?$
16. $(3x-12)\div(-3)=?$
17. $(15ab-12ac-9ax)\div(+3a)=?$

Exercise 62. Review of signed numbers

1. Make a number scale and show the following numbers: $+8$; $+3$; -6; -9; $+14$; -8; $+8-5$; $-8+5$; $-12-2$; $+4-6$; $+3+2-5$; $-7+9-3+5$; $-1-1+2+2-2$.

2. Find the average of the following thermometer readings: $+10°$, $+4°$, $-9°$, $-12°$, $-5°$, $0°$, $-1°$, $+3°$, and $+7°$.

3. The average monthly temperature of a place in Russia for a certain year was for January, $-42°$; for February, $-29°$; for March, $-8°$; for April, $+4°$; for May, $+18°$; for June, $+43°$; for July, $+58°$; for August, $+61°$; for September, $+38°$; for October, $+6°$; for November, $-8°$; for December, $-26°$. Find the average annual temperature.

4. What is meant by the absolute value of a number? Read the absolute value of -6; of $+18$; of the sum of -15 and $+3$; of the difference when $+9$ is subtracted from -4.

5. Give the sum, the difference, the product, and the quotient of -12 and $+3$; of $+6a$ and $-2a$.

6. $\dfrac{(-6)(+7)+(+4)(-6)}{(-7)-(+4)} = ?$

7. If given the date of a man's birth and the date of his death, how can you find his age at the time of his death? Use this plan to find the age at death of Augustus Cæsar who was born in the year -63 and died in the year $+14$.

8. How old at death was Cicero who was born in the year 106 B.C. and died in the year 43 B.C.?

9. Socrates was born 469 B.C. Confucius died 478 B.C. How long before the birth of Socrates did Confucius die?

10. What can be added to $-2x$ to get $-7x$? To $+3a$ to get $-5a$?

11. By what can $-x$ be multiplied to get $+x$? By what can $-x$ be divided to get $+x$?

2. What can be added to $x-3$ to get x? What can be led to $x+3$ to get x?

3. By what can $-\frac{1}{3}x$ be multiplied to get x?

4. What can be added to $2x-3$ to get -3? To -7 to 0?

5. $3-1-4-7=?$

6. $-a-a=?$

7. $(-a)(-a)+2a(-a)=?$

8. $-2^2-(-2)^2=?$

2. If $a=-4$ and $b=2$, find the value of $b-2a^2b$.

3. If $r=-1$ and $s=-2$, find the value of r^2-2rs; of $rs+r^2$.

4. $2ab-5ab-ab-3m+m=?$

5. $3a-2b-8a-b+3b-a=?$

6. $-x(x-2)=?$

8. $(-2rx+x-5x^2)\div(-x)=?$

9. $\dfrac{(-3)(-5)+4(-3)}{-3}=?$ Find in two ways.

0. $(x-2)(x+3)=?$

1. $(2n-1)(n-2)=?$

2. $(r-3s)^2=?$

3. $(x^2-2)(x^2-5)=?$

7. $(3x^2y-6xy^2+3xy)\div(-3xy)=?$

8. $(-m^3n^2+mn^3)\div(mn^2)=?$

9. $3(-2)^2-2(-3^2)=?$

0. $(3^3\cdot 2^2-2^3\cdot 3^2)\div(2^2\cdot 3^2)=?$

19. $x-2(-x)=?$

20. $x\div(-x)=?$

21. $-3^2\div(-3)^2=?$

34. $(a^2b-ab^2)(a-b)=?$

35. $(-5^3)(5^3)\div 5^2=?$

36. $(-3^2)(-3)^2\div(-3)^3=?$

CHAPTER VII

EQUATIONS AND PROBLEMS

57. Transposing terms in an equation.

The pupil has already learned how to solve simple equations and problems not involving negative numbers. He will find that many of the processes are made much simpler by using the knowledge of negative numbers which he now has.

What number added to $+4$ gives 0 as a result?

What number added to $-2x$ gives 0 as a result?

What number added to $2x-3$ gives $2x$ as a result?

What number added to $2x+3$ gives $2x$ as a result?

What number added to each member of the equation $3x-a=b$ gives $3x$ for the left member?

Add $-n$ to each member of the equation

$$2y+n=a.$$

Add $-2y$ to each member of the equation

$$2y+n=a.$$

Add $-a$ to each member of the equation

$$2y+n=a.$$

Observe that any term is eliminated, that is, removed from a member of the equation, by adding to both members that term with its sign changed.

Observe, too, that the term eliminated from one member appears in the other member with its sign changed.

Rule. *Any term may be changed from one member of an equation to the other provided its sign is changed.*

This process is called **transposition** and the term is said to be **transposed** from one member of the equation to the other.

EXAMPLE 1. Solve the equation $3x-7=5x-13$.

SOLUTION.	$3x-7=5x-13.$
Transposing $5x$ and -7,	$3x-5x=-13+7.$
Combining terms,	$-2x=-6.$
Dividing by -2,	$x=3.$
Checking,	$3\cdot 3-7=5\cdot 3-13.$
	$2=2.$

Steps in solving equations like $2x+5=7x-4$.

(1) Transpose the unknown terms to one member and the nown terms to the other.

(2) Combine like terms.

(3) Divide each member by the coefficient of the unknown.

(4) Check by substituting in the original equation the alue found for the unknown number.

EXAMPLE 2. Solve for x the equation, $5x+a=7a+3x$.

SOLUTION.	$5x+a=7a+3x.$
Transposing a and $3x$,	$5x-3x=7a-a.$
Combining terms,	$2x=6a.$
Dividing by 2,	$x=3a.$
Checking,	$15a+a=7a+9a.$
	$16a=16a.$

Exercise 63

Solve and check these equations.

1. $5x-2=18.$
2. $4x=x+12.$
3. $y=3y-12.$
4. $n-3=5n-23.$
5. $2r-7+3r+4=-2+4r.$
6. $7x=5x-13-2x-19.$
7. $2a-12=3a+17.$
8. $15n-7-11n=n-1+9.$
9. $\frac{1}{2}x+1=x+3.$
10. $1\frac{3}{4}x=\frac{1}{4}+x+1.$
11. $\frac{2}{3}=\frac{1}{2}n-1\frac{1}{3}.$
12. $4a-3-3a+4=0.$
13. $0=3x-5-5x-3.$
14. $.1a+.05=.01a.$
15. Solve for a, $a-bc=d$. Solve also for b; for c.
16. Solve for n, $mn-m^2=0$.
17. Solve for x, $(x-2)h=3h$.
18. Solve for h, $(x-2)h=3h+hx-5$.

58. Using equations to solve problems.

1. How many degrees are there in the sum of the angles of any triangle?

2. If told the sum of two angles of a triangle how may the third angle be found?

3. If told one of the acute angles of a right triangle how may the other acute angle be found?

4. If told one of the equal angles of an isosceles triangle how may the other two angles be found?

5. If told a person's age 5 years ago how can you find his age now? How can you find how old he will be 15 years from now?

6. If told the length and the width of a rectangle how can you find its area? Its perimeter? Its diagonal?

7. What is the next integer greater than n? The next smaller than n? Give five consecutive integers having 7 for the first; having 7 for the middle one; having n for the first; having n for the last; having n for the middle one.

Exercise 64

1. Twice a certain number is 10 more than the number. Find the number.

2. If 24 is taken from 3 times a certain number the result is 4 more than the number. Find the number.

3. The length of a rectangle is 2 inches more than its width. Its perimeter is 28 inches. Find its length and its width.

4. The length of a certain rectangle is 3 in. more than twice its width. Its perimeter is 6 in. more than 2 times its length. How long is it?

5. One of two children is 3 times as old as the other. In 2 years he will be just twice as old as the other. How old is each?

6. In 6 years a certain boy will be just twice as old as he was 4 years ago. How old is he?

7. The sum of 3 consecutive integers is 24. What are they?

8. The greatest of three consecutive odd numbers is 15 less than twice the sum of the other two. Find them.

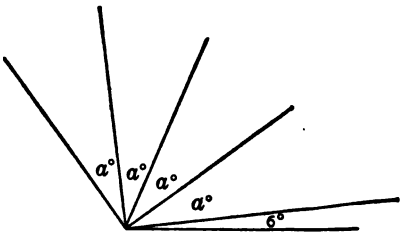


Fig. 70

9. An angle of 126° is formed by adding to an angle of 6° four equal angles, a. How many degrees in $\angle a$?

10. Find the value of angle x of Figure 71.

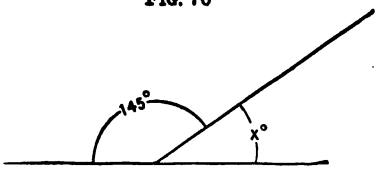


Fig. 71

11. The angle A is the complement of an angle of 54°. How many degrees in $\angle A$?

12. Three times $\angle X$ is the supplement of 24°. How many degrees in $\angle X$?

13. Find two complementary angles whose difference is 20°.

14. Find two supplementary angles whose difference is 36°.

15. How many degrees in the angle which is double its complement?

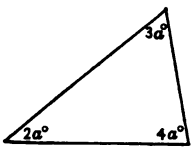


Fig. 72

16. How many degrees in the angle which is 10° more than $\frac{2}{3}$ of its supplement?

17. Find the number of degrees in each angle of the triangle of Figure 72.

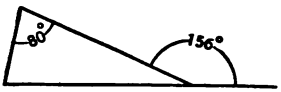


FIG. 73

18. Find the number of degrees in each angle of the triangle of Figure 73.

19. The vertical angle of an isosceles triangle is 24° less than the sum of the two equal angles. Find each angle of the triangle.

20. One of the acute angles of a right triangle is double the other. How many degrees in each angle of the triangle?

21. The supplement of a certain angle is 3 times its complement. How many degrees in the angle?

22. Write the equation stating that the supplement of a certain angle is double its complement. Solve it.

23. In Figure 74 find the value of $\angle x$ if $\angle C$ is a right angle. $\angle A$ is $3x$ and $\angle MBN$ is $2x$.

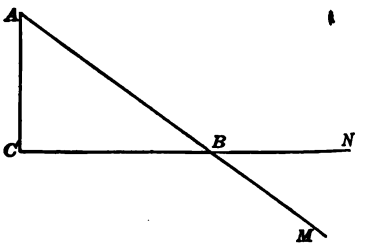


FIG. 74

24. How many sides has a polygon the sum of whose interior angles is 540°?

HINT. The formula for the sum of the interior angles of a polygon of n sides is $S=(n-2)180°$.

25. How many sides has the polygon the sum of whose interior angles is 1800°?

26. The circumference of a circle is 6 feet longer than its radius. Find its radius.

27. If the radius of a certain circle were increased 2 inches the circumference would then be 36 inches. What is the radius of the circle?

28. The diameter of one circle is 3 times the diameter of a second circle. The circumference of the first is 16π feet more than the circumference of the second. Find the diameter of each.

59. Equations involving fractions. The pupil will need to make use of the following rule for multiplication of fractions which he has learned: To multiply a fraction by an integer multiply the numerator or divide the denominator by the integer.

For example, $3\times\frac{2}{7}=\frac{6}{7}$; $3\times\frac{2}{3}=\frac{2}{3}$; $6\times\frac{5}{6}=5$.

What is the smallest number by which $\frac{3}{4}$ can be multiplied so that the product is an integer? What is the product?

Answer the same questions concerning $\frac{5}{8}$; $\frac{6}{5}$; $\frac{25}{7}$; $\frac{1}{1000}$; $\frac{35}{13}$.

Give a number by which $\frac{a}{b}$ can be multiplied so that the product is an integer, and give the product. Do the same for $\frac{x}{y}$; $\frac{5}{m}$; $\frac{m}{5}$; $\frac{a}{b^2}$; $\frac{a+b}{2}$; $\frac{5+x}{7}$.

What is the smallest number by which both $\frac{2}{3}$ and $\frac{1}{2}$ can be multiplied so that the two products are integers? What are the products? Answer the same questions concerning $\frac{3}{4}$ and $\frac{1}{2}$; $\frac{5}{6}$ and $\frac{2}{3}$; $\frac{5}{9}$ and $\frac{1}{3}$; $\frac{3x}{5}$ and $\frac{2x}{3}$; $\frac{1}{9}x$ and $\frac{5}{4}x$; $\frac{5}{7}a$ and $\frac{2}{5}a$. How are the numbers by which you multiplied found?

We shall need to perform such multiplications in solving equations containing fractions.

EXAMPLE 1. Solve for x, $\frac{x}{3}=12$.

SOLUTION. To get an equation that contains no fractions we multiply both members of $\frac{x}{3}=12$ by 3. This gives $x=36$.

Multiplying both members of an equation by a number so as to get an equation that does not contain fractions is called **clearing the equation of fractions.**

EXAMPLE 2. Solve for x, $\frac{x}{2}+\frac{x}{3}=15$.

SOLUTION. $\frac{x}{2}+\frac{x}{3}=15.$

Clearing of fractions by multiplying both members by 6, $3x+2x=90.$

Combining terms, $5x=90.$

Dividing both members by 5, $x=18.$

Check. $9+6=15.$

EXAMPLE 3. Solve for x, $\frac{2x}{3}-5=57-\frac{7x}{5}$.

SOLUTION. $\frac{2x}{3}-5=57-\frac{7x}{5}.$

Transposing and combining terms, $\frac{2x}{3}+\frac{7x}{5}=62.$

Clearing of fractions, $10x+21x=930.$

Combining terms, $31x=930.$

Solving for x, $x=30.$

Check. $20-5=57-42.$

$15=15.$

Exercise 65

Solve and check :

1. $\frac{x}{5}=7.$
2. $\frac{y}{2}=24.$
3. $\frac{n}{4}+6=2.$
4. $3+y=\frac{y}{2}.$
5. $\frac{5a}{3}-\frac{a}{4}=51.$
6. $\frac{2}{n}=16.$
7. $\frac{1}{n}=.4.$
8. $\frac{1}{x}=\frac{1}{6}.$
9. $\frac{7x}{10}-\frac{1}{2}=0.$
10. $\frac{x}{.75}-2=x.$
11. $\frac{a+6}{14}=1.$
12. $\frac{m-2}{4}=5.$
13. $.34-4x=1+2x.$
14. $.5y-.3=26+27.$
15. $\frac{r}{5}+\frac{r}{6}=11.$
16. $x-\frac{1}{2}=1-\frac{1}{2}x.$
17. $\frac{3m}{7}+5=\frac{m}{7}-5.$
18. $\frac{t}{100}+\frac{t}{10}+t=1.11.$
19. $\frac{3}{5}y-8=1-2y+.1.$
20. $\frac{2z}{3}+z=\frac{5z}{8}+3z.$

Exercise 66

1. If one-half of a number is added to five-sixths of the number the sum is 56. Find the number.

2. Two-fifths of a certain number is 3 more than one-third of the number. Find the number.

3. In a mixture of corn and oats there are $1\frac{1}{2}$ times as much corn as oats. There are 65 bu. of the mixture. How many bushels of each kind of grain?

4. In a mixture of cement, sand and crushed rock there are $2\frac{1}{2}$ times as much sand as cement and 4 times as much crushed rock as cement. Find how much there is of each kind of material in 300 cu. ft. of the mixture.

5. A 1 : 3 : 5 mixture of cement, sand and gravel is used in making the foundation of a storeroom. The foundation is 60 ft. by 38 ft., outside measurement, 7 ft. deep, and 15 in. thick. Find how much of each kind of material will be required to make this foundation.

6. What number must be added to the numerator of $\frac{4}{27}$ so that the resulting fraction equals $\frac{1}{3}$?

7. What number must be added to the denominator of $\frac{4}{27}$ so that the resulting fraction equals $\frac{1}{3}$?

8. What number added to the numerator and also to the denominator of $\frac{5}{9}$ gives a result equal to $1\frac{1}{3}$?

9. One kind of war bread contained $\frac{1}{4}$ as much potato flour as wheat flour. If a baker used at one time 140 lb. of the mixed flour, how much of each kind was used?

10. If to a number there be added $\frac{1}{2}$ of it, $\frac{1}{3}$ of it, and $\frac{1}{4}$ of it the result is 200. Find the number.

11. A line 36″ long is divided into two segments having the ratio of 4 : 5. How long is each segment?

12. A line is divided into two segments having the ratio 3 : 7. The longer segment is 28″. How long is the line?

13. The three sides of one triangle are 8′, 12′, and 15′. The longest side of a similar triangle is 20′. Find the other two sides of the second triangle.

14. In a ration of bran, oats, and oil meal the ratio is $3 : 2 : \frac{1}{4}$. How much of each in 30 bushels of the mixture?

15. Thirty per cent cream means cream that contains 30% butter fat. If a quart of 30% cream is mixed with a quart of 20% cream what is the per cent of cream in the mixture? If 3 qt. of 30% cream are mixed with 4 qt. of 20% cream what is the per cent of cream in the mixture?

16. How much 40% cream must be mixed with 4 qt. of 20% cream to get a mixture of 24% cream?

HINT. Let x = the number of quarts of 40% cream.
How many quarts of the mixture? How much butter fat in the 20% cream? How much butter fat in the x quarts of 40% cream? How much butter fat in the mixture? What two things are equal?

17. How much 25% cream must be mixed with 12 qt. of 40% cream to make a mixture of 35% cream?

18. How much 4.5% milk (milk containing 4.5% butter fat) must be mixed with 10 gallons of 2.4% milk to get 3.1% milk?

19. How much water must be added to a pint of a 10% solution of a certain medicine to reduce it to a 1% solution?

20. How much water must be added to a quart of 90% alcohol to reduce it to 60% alcohol? Ninety per cent alcohol means alcohol that contains 10% water.

60. Equations involving other letters besides the unknown. The pupil has already learned how to solve formulas for different letters. The following will give further practice with certain important formulas.

Exercise 67

Solve for x :

1. $mx=n$.
2. $x+m=n$.
3. $x-m=n$.
4. $m-x=n$.
5. $\frac{m}{x}=n$.
6. $\frac{x}{m}=n$.
7. Solve for y, $ay+by=c$.
8. Solve for t, $2t+vt=45$.
9. Solve for t, $t+vt=6$.
10. Solve for m, $am-m=80$.

11. Solve for r, $C=2\pi r$. Find the radius of a circle whose circumference is 132 in. In this and the following problems use $\pi=3\frac{1}{7}$, unless told otherwise.

12. Solve for a, $V=\pi r^2 a$. Find the altitude of a cylinder whose radius is 2 in. and whose volume is $100\frac{4}{7}$ cu. in.

13. The standard bushel is the volume of a cylinder $18\frac{1}{2}$ in. in diameter. Find its height. Use $\pi=3.1416$.

14. Solve for r, $A=2\pi ra$. Find the radius of a cylinder whose lateral surface is 3520 sq. in. and whose altitude is 35 in.

15. Solve $C=\pi d$ for d. Find the diameter of a circle whose circumference is $8\frac{1}{2}$ ft.

16. A piece of wire 4 ft. 6 in. long is bent into the form of a circle. Find the diameter of the circle.

17. Solve $C=\frac{5}{9}(F-32)$ for F. Find the centigrade temperature, C, when the Fahrenheit temperature, F, is $-22°$. Find the Fahrenheit temperature when the centigrade temperature is 29°.

18. Solve $S=(n-2)180$ for n. Find the sum of the angles of a polygon of 12 sides. Find the number of sides of a polygon if the sum of the angles is 2520°.

19. Solve for n, $A=\frac{(n-2)180}{n}$. Find the number of degrees, A, in one of the angles of a regular polygon of 10 sides. Find the number of sides of a regular polygon if the number of degrees in one of the angles is 90; also if the number of degrees in one of the angles is 135.

20. In the set of numbers 3, 8, 13, 18, 23, . . . , each number is 5 greater than the preceding number. In the set of numbers a, $a+d$, $a+2d$, $a+3d$, $a+4d$, . . . , each number is d greater than the preceding number. It can be proved that if there are n such numbers in a set of which a is the first and l is the last, then $l=a+(n-1)d$. Find l if $a=6$, $n=10$, and $d=4$. Find a if $l=70$, $n=13$, and $d=7$.

21. Solve the formula in the preceding problem for d and find d if $l=2396$, $a=-4$, and $n=25$.

22. Solve the same formula for n and find n if $a=0$, $l=88$, and $d=8$.

23. A bookkeeper accepts a position at a salary of \$900 a year. If his salary is increased \$75 each year what will be his salary the tenth year?

24. If a body falls 16 ft. the first second, 48 ft. the second second, 80 ft. the third second, and so on, how far will it fall the tenth second?

25. A ball rolls down an inclined plane at the rate of 3 ft. the first second, 7 ft. the second second, 11 ft. the third second, and so on. In what second will the ball be rolling at the rate of 43 ft. a second?

61. Equations containing the square of the unknown. The formula for finding the area of a square of side s is $A = s^2$. This equation may be solved for s by taking the square root of both sides of the equation, which gives $\sqrt{A} = s$, or $s = \sqrt{A}$. This gives a formula for finding the side of a square when the area is given. For example, if $A = 625$, $s = \sqrt{625} = 25$.

There are other useful formulas which we shall wish to solve and which involve the square of the unknown.

Exercise 68

1. Solve $r^2 = k$ for r. Find r when $k = 169$; when $k = .0256$.

2. Solve $y^2 = 2px$ for y. Find y if $p = 6$ and $x = 12$; also if $p = .9$ and $x = 20$.

3. If $x^2 + y^2 = r^2$, show that $x = \sqrt{r^2 - y^2}$. Solve the equation for y. Solve it for r. Find x if $r = 14$ and $y = 12$.

4. Show that x, y, and r in the preceding exercise may represent the three sides of a right triangle.

5. What is the diagonal of a rectangle whose sides are 25 yd. and 60 yd.?

6. A rope that is known to be 80 ft. long is attached to the top of a pole. When the rope is stretched it touches the ground 30 ft. from the foot of the pole. Find the height of the pole.

7. Solve $A = \pi r^2$ for r. Find the radius of a circle whose area is 616 sq. ft. Use $\pi = 3\frac{1}{7}$.

8. What must be the radius of a circular flower bed to contain the same area as a rectangular one which is 6 ft. by 16 ft.? Use $\pi = 3\frac{1}{7}$.

9. The area of the State of New York is 49,204 sq. mi. Find the radius of a circle of the same area, correct to .1 mi. Use $\pi = 3.1416$.

10. The formula $s = 16.1\,t^2$ gives the number of feet, s, that a body will fall in t seconds if it starts from rest. (This formula does not take into account the resistance of the atmosphere.) Solve this formula for t. Find t if $s = 788.9$ ft.

11. Find the number of seconds required for a body to fall 1000 ft.

12. Find how many seconds are required for a bomb to fall 1 mile.

13. Solve $V = \pi r^2 a$ for r. Find the radius of a cylinder whose volume is 2376 cu. in. and whose altitude is 21 in.

14. A quart cup is to have a diameter of 4 in. What must be its altitude? A quart is 57.75 cu. in.

15. Solve $S = 4\pi r^2$ for r. Find the radius of a sphere whose surface is $314\frac{2}{7}$ sq. in.

16. What must be the diameter of a sphere to have a surface equal to the surface of a cube whose edge is 6 in.? Before computing the answer make an estimate of what the answer will be.

17. Write a formula for finding the total surface of a cube of edge e. Solve this formula for e. Find the edge of a cube whose total surface is 661.5 sq. ft.

18. The area of an equilateral triangle of side s is given by the formula $A = .433s^2$. Solve this equation for s. Find the side of an equilateral triangle whose area is 100 sq. in. Find the answer correct to .1 in.

CHAPTER VIII

GRAPHS

62. The use of graphs. The pupil has learned in the First Book how to represent a number by a straight line. He has learned also that the line is called the **graph** of the number.

There are other forms of the graph besides the straight line. So much use is made of graphs in illustrating articles in newspapers, magazines, and books that it is important for the pupil to learn how to make graphs and how to interpret them.

EXAMPLE. In a certain school an examination in arithmetic is given each month. The examination consists of 10 questions. One boy made the following record. The first month he answered 8 questions; the second month, 9; the third month, 7; the fourth month, 10; the fifth month, 9; the sixth month, 8; the seventh month, 10; the eighth month, 9; the ninth month, 10. These numbers are represented by the graph in Figure 75.

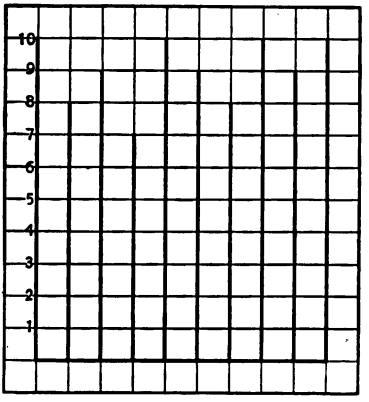


FIG. 75

In representing numbers by lines, as in Figure 75, or by a bar graph, as in Figure 76, it is convenient to use **cross-ruled paper**. This paper should be used by pupils in making graphs. It may be obtained from any stationer.

EXAMPLE 2. The average monthly rainfall in Seattle, Washington, is given in the following table:

JAN.	FEB.	MARCH	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
4.5	3.9	3.6	2.7	2.3	1.6	0.6	0.4	1.8	2.9	5.9	6.2

The variation of the monthly rainfall is well shown in the bar graph in Figure 76. One space represents one inch of rainfall.

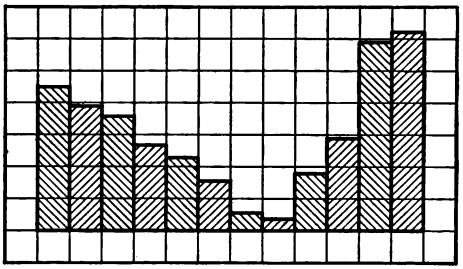


FIG. 76

Exercise 69

1. In ten trials of ten shots each a marksman makes the following scores: 6, 7, 7, 9, 8, 7, 8, 8, 9, 8. Represent this by a graph as in Figure 75.

2. The average monthly rainfall in New Orleans is given in the following table. Represent it by a bar graph.

JAN.	FEB.	MARCH	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
4.6	4.4	5.3	4.9	3.9	6.3	6.5	5.6	4.8	2.9	3.9	4.5

Compare the graphs of the monthly rainfall in Seattle and in New Orleans. In what season is the rainfall greatest in Seattle? In New Orleans? In what season least in each city? In what month is the rainfall least in each city? Greatest?

3. The following scores were made by some eighth grade pupils in two tests in addition given several weeks apart:

Pupil's number . . .	1	2	3	4	5	6	7	8
First test	34	35	47	48	48	52	55	58
Second test	43	41	55	65	66	70	67	71

Make a bar graph of these scores that will show the pupils' improvement. Let a score of 10 be represented by one space on the cross-ruled paper.

4. The number of one hundred million bushels of corn grown in the United States for ten years was as follows:

1908	1909	1910	1911	1912	1913	1914	1915	1916	1917
27	26	29	25	31	24	27	31	26	32

Represent these numbers by a graph.

5. The average selling price in cents per pound of sugar and of bacon in the United States on August 15 each year for 5 years is given in the following table. Represent these prices by bar graphs. Which increased faster, the cost of sugar or the cost of bacon?

Year	1913	1914	1915	1916	1917
Average cost of 1 lb. of sugar	5.6¢	7.8¢	6.7¢	8.5¢	9.9¢
Average cost of 1 lb. of bacon	28.1¢	28.7¢	27.0¢	29.3¢	43.0¢

Figure 77 gives the graph of temperatures from 6 A.M. to 6 P.M. Notice that 3 spaces along the horizontal line OX represent 1 hour, and that 1 space along the line OY represents 1 degree. OX is used as the **axis of time** and OY as the **axis of temperature**. What was the temperature at nine

o'clock? At one o'clock? At four o'clock? At what hour was the temperature 76°? At what hour was the tempera-

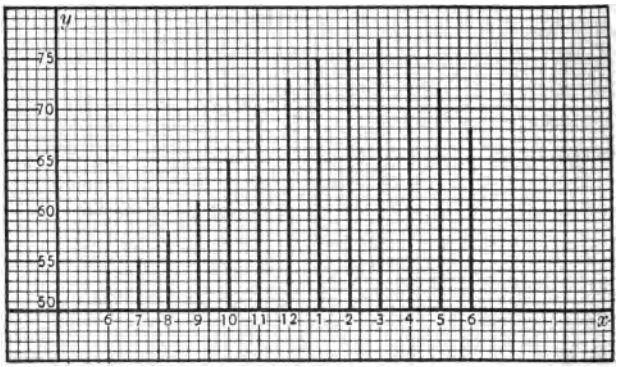


FIG. 77

ture 55°? What do you think the temperature was at half-past eight? At half-past two?

Let the pupil make a copy of this graph on a larger scale. Draw in the lines to represent the temperature at each half-

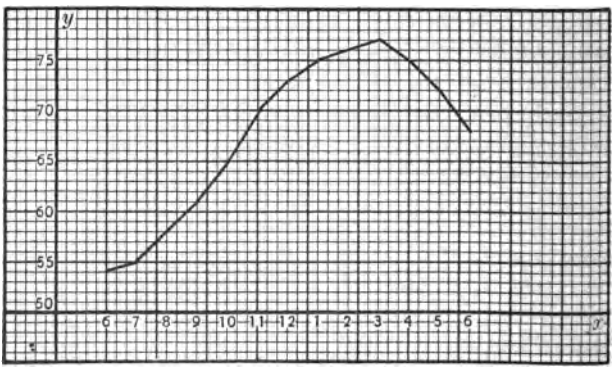


FIG. 78

hour and at each quarter-hour. Suppose now that lines were drawn to represent the temperature at every five

minutes, or even at every minute. If the tops of these lines were connected what kind of figure would be formed?

If now the pupil will take another sheet of cross-ruled paper, and instead of drawing the lines to represent the temperatures, will mark in only the upper ends of these lines and then join these points by a smooth curve, he will get a curve like Figure 78. This curve is another kind of graph, and is one that is much used.

Exercise 70

1. Figure 79 gives the graph of temperatures from midnight to noon. Notice that temperatures below zero are measured downward from the axis of time. What is the temperature at four o'clock? At six o'clock? At nine

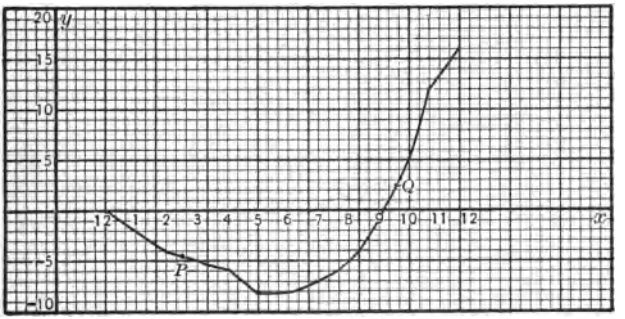


FIG. 79

o'clock? At what hour is the temperature $-4°$? $10°$? $-3°$? Find the time and the temperature at the point P on the curve; at the point Q.

2. The following table gives the hourly temperature for a first day of February in Chicago. Make the temperature curve as in Figure 79.

Hour	Temperature	Hour	Temperature	Hour	Temperature	Hour	Temperature
2 A.M.	0	8 A.M.	−3	2 P.M.	10	8 P.M.	8
3 A.M.	−1	9 A.M.	1	3 P.M.	11	9 P.M.	8
4 A.M.	−2	10 A.M.	1	4 P.M.	10	10 P.M.	8
5 A.M.	−3	11 A.M.	6	5 P.M.	9	11 P.M.	7
6 A.M.	−4	12 M.	8	6 P.M.	9	12 P.M.	7
7 A.M.	−4	1 P.M.	9	7 P.M.	9	1 A.M.	6
7:30 A.M.	−5						

3. Observe and record the temperatures for 8 hours and make the graph.

4. Life insurance companies use tables which give the "expectation of life" of persons at different ages, that is, the average number of years that persons of a given age live beyond that age. For example, in the table here given the expectation of life of a person 20 years old is 43 years, which means that a person 20 years old may expect to live 43 years longer. What age may a person 15 years old expect to reach? A person 70 years old?

Age	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85
Expectation of life	55	51	47	43	39	35	31	27	24	20	17	14	11	9	7	5	4

Make a graph to show the expectation of life as shown in this table. Find your own expectation of life. Find the expectation of life of a person 18 years old; of a person 32 years old; of a person 58 years old. How old is a person whose expectation of life is 45 years? 16 years?

5. This graph, Figure 80, shows the cost of a number of yards of cloth at $2 a yard. Two spaces on the horizontal axis represent 1 yard and two spaces on the vertical axis represent $1. Show on the graph how to find the cost

of 8 yards ; of $3\frac{1}{2}$ yards ; of $6\frac{1}{2}$ yards. Show how to find how many yards can be bought for \$7 ; for $\$3\frac{1}{2}$; for $\$1\frac{1}{2}$.

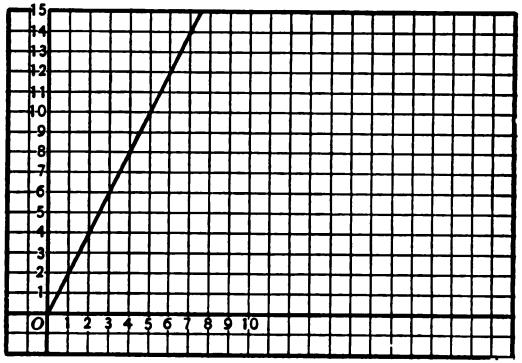


FIG. 80

6. Make a graph to show the cost of shoes at \$4 a pair.

7. Make a graph to show the interest on \$100 at 6% for each year up to 10 years. From this graph show how to find the interest for $3\frac{1}{2}$ years; for 4 years 3 months. Show how to find the time when the interest equals \$30 ; \$48 ; \$52.

8. Make a graph to show the interest on \$1 at 5% for each number of years from 1 to 10.

9. One of the most frequent applications of graphs is in finding the graphs of equations. Figure 81 gives the graph of the formula $A = e^2$. Find from the graph the square of 3 ; of $4\frac{1}{2}$; of 3.4. Find from the graph the number of which 20 is the square ; of which 30 is the square ; of which 45 is the square.

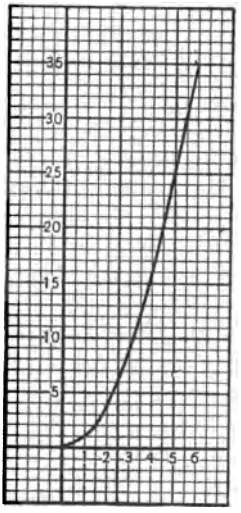


FIG. 81

10. Make a graph to show the distance that a body starting from rest falls in t seconds, as given by the formula $S = 16.1t^2$. First fill out this table.

t	1	2	3	4	5	6
S						

Find from the graph how far a body falls in $2\frac{1}{2}$ seconds; in $3\frac{1}{2}$ seconds. How many seconds does it take a body to fall 100 ft.? 200 ft.? 250 ft.?

11. Make a table as in Exercise 10 to show the values of y in the formula $y = 3x - 2$ when x has the values -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5.

Make a graph. On this graph find the value of y when x is -3.5; when x is .8. What is the value of x when y is 6? When y is 0? When y is -4?

12. The cost of sending a package by parcel post depends upon the weight of the package and the distance it is to be sent. The country is divided into zones according to the distance from the starting point. The cost of sending a five-pound package to each of the zones is given in the following table. Make a bar graph showing the cost.

Zone	1st	2nd	3rd	4th	5th	6th	7th	8th
Cost in cents	9	9	14	23	32	41	51	60

13. The number of games won by the different teams in the American League in a certain year was as follows:

Chicago	100	Washington	74
Boston	90	New York	71
Cleveland	88	St. Louis	57
Detroit	78	Philadelphia	55

Make a bar graph showing the number of games won by each team.

14. According to a report of the United States Department of Agriculture the average yield of wheat in bushels per acre for a period of ten years in five countries was as follows :

Great Britain . .	33.4	Hungary . . .	18.1
Germany . . .	30.7	United States . .	14.8
France	20.1		

Show this by a graph.

Exercise 71. The Money Value of Education *

1. In a certain year Massachusetts spent an average of $38.55 per pupil for the education of the children of the state. In that year Tennessee spent an average of $4.86. In that year the citizens of Massachusetts earned on the average $466 each, and those of Tennessee $174 each. What was the difference in cost to the state in these two states of sending a child to school for eight years? How much more would an average citizen of Massachusetts earn in forty years than an average citizen of Tennessee?

2. Investigations in Brooklyn, New York, and in Springfield, Massachusetts, show that in those cities citizens who are high-school graduates earn on the average about $1000 a year, and those who are not high-school graduates about $500. Using these figures, how much more will a high-school graduate earn in 40 years than a non-high-school graduate? This amounts to how much a day for each day that the high-school graduate has spent in high school, assuming that a high-school graduate has been in high school 4 years of 180 days each?

* The data for these problems were taken from Bulletin, 1917, No. 22: The Money Value of Education, published by the United States Bureau of Education.

3. In this graph, Figure 82, the solid black columns represent the average yearly wage of 584 persons who left school at 14 years of age. The hatched columns represent the average wage received by 215 persons who remained in

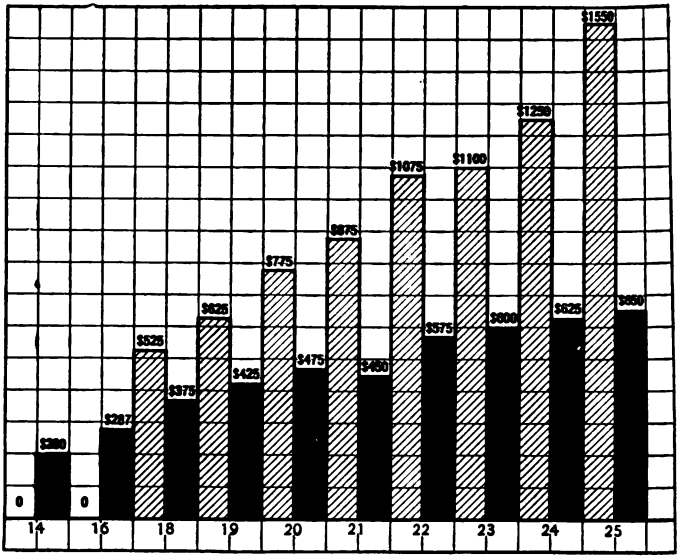


FIG. 82

technical schools till 18 years of age. Find the average amount received by a person in each group from the age of 14 to the age of 25 inclusive. The figures in this graph were taken from a report of a Commission on Industrial and Technical Education in Massachusetts.

4. From a study of a group of distinguished Americans it was found that of 5,000,000 American men with no schooling, 31 had attained distinction; of 33,000,000 with elementary education only, 808 had attained distinction; of 2,000,000 with high school education, 1245 had attained distinction; of 1,000,000 with college education, 5768 had attained dis-

tinction. We may say then that a man with no schooling has 31 chances out of 5,000,000 of attaining distinction, or a chance of .0000062. Find the chance of attaining distinction of a man in each of the other classes. Express the answers as decimals. The chance of the college graduate to attain distinction is how many times as great as the chance of each of the other classes?

5. This table gives a comparison of the wages of persons who left New York City schools at 14 years of age with those who left at 18 years of age.

Weekly Salary at	Left School at 14	Left School at 18
14 years of age	$ 4.00	0
15 years of age	4.50	0
16 years of age	5.00	0
17 years of age	6.00	0
18 years of age	7.00	$10.00
19 years of age	8.50	10.75
20 years of age	9.50	15.00
21 years of age	9.50	16.00
22 years of age	11.75	20.00
23 years of age	11.75	21.00
24 years of age	12.00	23.00
25 years of age	12.75	31.00

Find the average amount received by a person of each class from the age of 14 to the age of 25 inclusive. Find the difference in the annual salaries in the two groups at the age of 25. Make a graph similar to Figure 82. What is the per cent of increase yearly in each case from 18 to 25 years?

CHAPTER IX

APPLICATIONS OF PERCENTAGE

63. Use of percentage in business. The principles of percentage which the pupil has already learned are applied to many difficult problems of business, some of which the pupil is now ready to understand.

All of us will have business with banks and should understand how their problems are solved. We shall have to pay taxes and shall want to know how they are computed. We shall want to send money to distant places, shall want our property insured, and shall meet many other problems to which the principles of percentage must be applied.

The pupil will find no new principles of percentage involved in these problems. Their difficulty consists in the business conditions involved, with which the pupil has had little experience. To understand clearly the problems of this chapter the pupil should be given experiences as nearly like those in real business transactions as possible. He should write checks, give and receive notes, buy drafts, compute taxes and go through the forms of paying them. His class can organize a stock company to buy a farm or manage a bank, and each member of the class can have a part in the various business transactions involved.

64. The percentage formulas. The problems which follow require the pupil to know the percentage formulas.

Find $33\frac{1}{3}\%$ of 360. Name the base, rate, and percentage.

Give the formula for finding the percentage when base and rate are known; the rate when percentage and base are known; the base when percentage and rate are known.

Exercise 72

1. Find the term not given.

Base . . .	\$960	800 yd.	1225	?	\$8.40	?
Rate . . .	3%	?	16%	20%	?	.1%
Percentage .	?	32 yd.	?	8	\$125	\$2.86

2. Express as per cents : .12, .08, 5.67, 1.00, 10.00, 9, .06⅔, .009, .3468.

3. State a rule for changing decimals to per cents.

4. Express as decimals : 6%, 75%, 3.6%, .08%, 346%, 100%, 842.9%, ½%, ⅞%.

5. State a rule for changing per cents to decimals.

6. Express as per cents : $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{12}$, $\frac{3}{20}$, $\frac{4}{25}$.

7. Express as common fractions in their lowest terms : 5%, 25%, 60%, 75%, 12½%, 16⅔%, 33⅓%, 140%, 65%, 48%, 66⅔%, 137½%.

8. The enrollment in a school was 550 pupils in a certain year. The following year the enrollment decreased 18%. What was the enrollment the second year?

9. A teacher's salary is increased from \$75 a month to \$80 a month. What is the per cent of increase?

10. Find a number that is 10% more than 960.

11. Find a number that is 6% less than 75.

12. 165 is 10% more than what number?

13. 380 is 5% less than what number?

14. The corn crop of the United States in a certain year was 2,566,927,000 bu., and in the following year 3,159,494,000 bu. What was the per cent of increase?

15. At the same time the wheat crop increased from 636,318,000 bu. to 650,828,000 bu. What was the per cent of increase in the wheat crop?

16. On a certain day the price of wheat in the Chicago market ranged from \$2.14 a bushel to \$2.46 a bushel. The second price is what per cent of the first? The first price is what per cent of the second? What profit would have been made by a man who bought 10,000 bu. at the lower price and sold it at the higher? What per cent of the cost?

17. A dairyman sold 2500 lb. of milk which averaged 2.8% butter fat. That was how many pounds of butter fat? What was the butter fat worth at 24¢ a pound?

18. In 1917 the price of silver advanced from 78¢ to \$1.08½ an ounce. What was the per cent of increase in price?

19. A farmer invests \$24,000 in a farm. The first year it pays him 4½% profits. Find the amount of profits.

65. Useful equivalents. Much use is made in percentage of the following equivalents. The pupil should practice until these equivalents can be given in 60 seconds or less.

$25\% = \frac{1}{4}$.	$40\% = \frac{2}{5}$.	$87\frac{1}{2}\% = \frac{7}{8}$.	$83\frac{1}{3}\% = \frac{5}{6}$.
$50\% = \frac{1}{2}$.	$10\% = \frac{1}{10}$.	$62\frac{1}{2}\% = \frac{5}{8}$.	$14\frac{2}{7}\% = \frac{1}{7}$.
$75\% = \frac{3}{4}$.	$80\% = \frac{4}{5}$.	$16\frac{2}{3}\% = \frac{1}{6}$.	$11\frac{1}{9}\% = \frac{1}{9}$.
$20\% = \frac{1}{5}$.	$12\frac{1}{2}\% = \frac{1}{8}$.	$33\frac{1}{3}\% = \frac{1}{3}$.	$6\frac{1}{4}\% = \frac{1}{16}$.
$60\% = \frac{3}{5}$.	$37\frac{1}{2}\% = \frac{3}{8}$.	$66\frac{2}{3}\% = \frac{2}{3}$.	$8\frac{1}{3}\% = \frac{1}{12}$.

Exercise 73

1. Find these per cents of the numbers in the first column:

	10%	25%	12½%	37½%	16⅔%	83⅓%
200						
450						
12						
7.5						
3460						

2. A grocer buys cheese at 26¢ a pound ; lard at 28¢ a pound ; sugar at $7.95 per hundred pounds ; and canned salmon at $2 per dozen cans. He intends to sell so as to make 10% of the cost. Find the selling prices, correct to the nearest cent.

3. In a factory the wages per hour for four different grades of employees were as follows: 80¢, 65¢, 55¢, 42¢. All of these wages were increased $8\frac{1}{3}\%$. Find the resulting hourly wages, correct to the nearest cent.

4. Find numbers of which the following are $12\frac{1}{2}\%$: 8.2, 45, 921, .065, $\frac{3}{4}$, $\frac{1}{10}$.

5. Find numbers of which the following are $16\frac{2}{3}\%$: 16, 4.8, .003, $\frac{5}{6}$, $3\frac{1}{2}$, .7.

6. Find numbers of which the following are 75% : 60, 300, .018, $\frac{6}{7}$, $\frac{1}{2}$, .05.

7. Find numbers of which the following are $66\frac{2}{3}\%$: 28, 420, .08, .006, 64.25, .12.

8. A wholesale merchant sold $45,000 worth of goods during a certain year. The next year his sales increased 25% and the year following his sales decreased 20%. What were his sales the third year?

9. During his first year of employment a clerk gets $75 a month. During the second year his monthly salary is 10% more than during the first year, and during the third year 10% less than the second year. Find his monthly salary during the second and third years.

10. A number is increased 50%, and then this result decreased 50%. Is the final result the same as if the number had been decreased 50% and this result increased 50%? Is the final result the same as the original number in either case?

11. A number is increased 50%, then this result is increased $33\frac{1}{3}\%$, and then this result increased 50%. By what per cent is the original number increased each time?

66. Profit and loss based on the initial cost. To make a success of any business it is necessary to keep an account of the amount invested, the expenses, the receipts, and profits and losses. In many cases the rate of profit or loss is computed as a per cent of the initial cost, without considering other expenses.

Exercise 74

1. A dealer bought a horse for $250 and sold it at a profit of 18%. Find the profit and the selling price.

2. Find the selling prices of the following articles which are sold at 20% above cost :

Article	Cost	Article	Cost
Men's shoes . . .	$ 3.75	Shirts . . .	$ 1.75
Hats	2.25	Boys' suits . .	15.00
Overcoats . . .	19.50	Gloves . . .	2.50
Ties	.75	Raincoats . .	16.25

3. On a certain day during the World War the United States Food Administration published the following buying and selling prices for retailers of food :

Article	Buying Price	Selling Price	Article	Buying Price	Selling Price
Eggs, per doz.	38¢	43¢	Butter . .	42¢	52¢
White bread, 1 lb. loaf . . .	8¢	9¢	Potatoes .	$1.50 per 100 lb.	18¢ per lb.
Hams, per lb. .	30¢	35¢			
Sugar, granulated	$7.87 per 100 lb.	8½¢ per lb.	Flour, ¼ bbl.	$2.75	$3.00
			Chickens per lb.	33¢	38¢

Find the per cent of profit allowed the retailer in each case, correct to .1%.

4. A department store advertises the following :

Suits formerly sold for \$75, now \$25. Coats formerly sold for \$75, now \$35. Coats formerly sold for \$50, now \$25. Gowns formerly sold for \$95, now \$35.

Find the per cent that each former selling price has been reduced.

5. A house was sold at a profit of 20%. The selling price was \$4800. Find the cost.

6. A dealer sells two books for \$3 each. On one he gains 20% and on the other he loses 20%. Find the total gain or loss.

7. A merchant buys shoes at \$42.50 a dozen pairs and sells them at \$4.50 a pair. Find his per cent of gain on the cost.

8. The owner of a restaurant buys a 12-pound ham for 32¢ a pound. It loses 30% of its weight when baked. For what must he sell it a pound to gain 15%? Find the answer to the nearest cent.

9. Sold $\frac{2}{3}$ of a sack of coffee for $\frac{1}{3}$ of the cost of the whole sack. Find the rate per cent of gain.

10. Some merchants compute the rate of profit or loss as a per cent of the selling price. Using this plan compute the rate of profit on each article in exercise 3, correct to .1%.

67. Profit and loss based on total cost. The merchant who bases his per cent of profit on the initial cost of each article is very apt to become bankrupt while appearing to be making a good profit. If a man buys a horse for \$100, keeps him for 6 months and then sells him for \$125, it appears that he has made a profit of 25% on the horse. But if he has fed the horse for the 6 months at a cost of \$24, and has paid \$7 for his care, the man has had a total investment of \$131 in the horse and has a net loss of \$6.

The merchant who buys a boy's suit for \$8.25 and sells it

for \$12 should add to the initial cost of the suit the cost of selling it before computing his profit. This cost of selling includes such items as clerk hire, rent of building, interest on capital, and other business expenses.

These expenses for carrying on a business are called **overhead expenses** or **overhead.**

EXAMPLE. Suppose that the initial cost of an article is \$50 and that the overhead expenses charged against it are \$3.50. Then a gain of 20% means a gain of 20% of \$53.50, or \$10.70.

Exercise 75

1. A motor boat cost the dealer \$380. Overhead expenses for selling it are estimated to be \$40. What must be the selling price to gain 15%?

2. A retail grocer estimates that his overhead expenses are 6% of the initial cost of the goods he sells. Find the selling price of the following if he gains 10% on the sum of the initial cost and the overhead. Fill out this table :

ARTICLE	INITIAL COST	OVERHEAD	SELLING PRICE
Bacon . . .	35¢ per lb.		
Navy beans .	\$12 per 100 lb.		
Rice	\$8.24 per 100 lb.		
Cheese . . .	24¢ per lb.		
Corn meal . .	\$4.25 per 100 lb.		

3. A merchant finds at the end of the year that the cost of goods sold during the year has been \$257,438. The overhead charges are \$40,896. His net profits are \$28,942. Find the rate of profit on the amount expended.

4. The automobile department in a department store uses \$125,000 capital. It is charged with \$18,850 overhead. This department is expected to show a profit of 12½% on the capital and overhead. What amount of profits is expected?

68. Commission. Some men are employed to buy or to sell goods for others. More frequently it is to sell goods or other property. Traveling salesmen who go from town to town selling goods for the wholesale houses are of this class. Others sell live stock for the farmers and stock dealers to the packing companies, and many others sell fruit and vegetables for the farmers to the grocers of the cities. These men are not usually paid for their services by the day but are paid a certain per cent of the selling price of the articles sold.

Pay computed in this way is called **commission** and the men who receive it are generally known as commission men.

A farmer who has a carload of hogs to sell may ship them to the city to a commission man. The commission man sells them to the packing company, receives the pay for them, deducts a certain per cent of the selling price as his pay or commission, and sends the farmer the balance.

Exercise 76

1. If you were to sell popcorn at the fair on a commission of 10%, how much would you be paid for selling 754 bags of popcorn at 5¢ each?

2. If you could sell 320 bags a day at 5¢ each and get a commission of 8%, would you earn as much as $1.25 a day?

3. Mr. Henry ships a carload of hogs to a commission man. The commission man sells them to a packing company for $6.75 a hundred pounds and charges 1% commission. The hogs weigh 14,700 pounds. How much is the commission?

4. The freight charges on a carload of cattle were $1.25 per hundred pounds. The commission firm charged $\frac{7}{8}$% commission for selling them. The cattle weighed 24,280 pounds when sold, but had lost 235 pounds in shipment. The shipper paid $7.50 a hundred for them and the commission firm sold them for $8.45 a hundred. How much did the shipper make?

5. A berry grower ships 127 crates of berries to a commission man, who sells them to grocers at $3.50 a crate. He charges a commission of 2%. How much does he remit to the berry grower?

6. A hardware salesman finds that his sales for a week amount to $960. If his commission is 2%, besides a fixed salary of $40 a week, how much has he earned this week?

7. How many bags of popcorn at 5¢ each would you have to sell on a 12% commission to earn as much in a week as you would earn on wages of a dollar and a half a day?

8. John agrees to pick apples for his father on a commission of 5% of the apples he picks. At the end of the first day he has picked 30 bushels. How many should he keep? If the apples are all sold at 75¢ a bushel, and John received 5% of the selling price, would he be as well paid as to receive 5% of the apples?

9. A book agent gets a commission of 35% for selling a set of books which he sells for $28 a set. If his average sales are 3 sets a week, how much are his profits in 10 weeks?

10. A real estate dealer gets $450 for selling 120 acres of land at $250 an acre. Find his rate of commission.

11. A traveling salesman gets a salary of $150 a month. In place of this salary he is offered a 5% commission on his sales. What must be the amount of his sales to bring him an income equal to his present salary?

12. A student spent his summer vacation selling aluminum kitchen utensils and a cleaning preparation. His rate of commission was 35% on the utensils and 45% on the cleaning preparation. He sold $985 worth of the utensils and $78 worth of the preparation during the summer. How much did he earn that summer? In July alone he sold $445 worth of utensils and $35 worth of the preparation. How much did he make in July?

69. Taxes. All the people of a community are benefited by its schools, its roads, the city lights, and the police protection. All the people should help pay for them, but since some people are more able to pay for them than others, each person is required to pay in accordance with the value of his property.

An estimate is made of the value of each person's property by an officer called the **assessor.** The value placed upon property by the assessor is called its **assessed valuation.** In most states this is much less than its real value.

Mr. Perkins lives in a school district in which the assessed valuation of all the property is \$60,500. The school needs for the year's expenses \$1512.50. This is $2\frac{1}{2}\%$ of the assessed valuation. The assessed valuation of Mr. Perkins' property is \$2400. He should therefore pay $2\frac{1}{2}\%$ of \$2400 as his share of the school expenses. This is called his school **tax.**

The per cent which is paid for schools is added to that which is paid for roads and all other purposes, and this total per cent is called his **tax rate.**

The tax rate for a particular purpose is usually less than 1% and is therefore usually expressed as so many mills on the dollar.

Exercise 77

1. When is the property assessed in your district?
2. Who fixes the value of the property?
3. Who collects the taxes? How often?
4. What was the tax rate in your district last year?
5. If the school tax rate in a certain district is 1.2%, how much tax is paid by Mr. Walters whose assessment is \$675? By Mrs. Bond whose assessment is \$1175? By Mr. Adams whose assessment is \$420? By Mr. Elkins whose assessment is \$2400? By Mr. Grindle whose assessment is \$1560?
6. The total assessed valuation in Alamo is \$5,400,000. What tax rate must be levied to meet the city's annual expenses of \$81,000?

7. How much city tax must be paid by Mr. Roberts of Alamo if his property is assessed at $8750? By Mrs. Anderson, assessed at $2700?

Find the property tax on the following :

8. Assessed valuation $4850, tax rate 1.5%.

9. Assessed valuation $56,784, tax rate 2.8%.

10. Assessed valuation $1,485,200, tax rate 2.75%.

11. Assessed valuation $2,003,875, tax rate 1.9%.

Find the tax rate, correct to .01%, if the following amounts of taxes are paid on the given valuations :

12. Valuation $875, amount of tax $21.

13. Valuation $3750, amount of tax $67.50.

14. Valuation $2,795,640, amount of tax $44,730.24.

15. In computing the cost of raising an acre of grain it is necessary to add to the cost for labor and the cost for seed, the cost for rent and taxes. Estimating the value of a certain acre of land at $125, its rental value as 5% of that amount, and the tax rate 2½% of an assessed valuation of three-fifths of the real value of the land, what is the cost of raising an acre of wheat if the labor and machinery cost $5.64 and the seed $1.50?

16. If the acre of the preceding problem yields 23 bushels of wheat, which is sold at $1.20 a bushel, what is the per cent of profit on the total cost of production?

17. A farm of 80 acres is valued at $11,200, but is assessed for taxation at $8500. The net income from crops is 7% of the valuation. The tax rate in the district is 2.25%. Will the owner's income be more or less if he sells the farm for $11,200 and lends the money at 7½%, paying the taxes on the whole amount of the loan?

70. Government revenues. The expenses of the United States Government are paid by revenues which are obtained mainly from tariff or customs duties; internal revenues on tobacco, and certain other articles; other stamp taxes; and the income tax.

71. Customs duties. Tax is paid on many kinds of imported goods. This is called **customs duties** or **tariff**.

Customs duties are of two kinds, **specific** and **ad valorem**.

A **specific duty** is reckoned on the quantity of the goods, as so much per gallon or pound. An **ad valorem duty** is a certain per cent of the value of the goods in the country in which they are purchased.

The following gives the duties on certain articles according to a certain tariff law.

Automobiles, valued at $2000 or more	45% ad valorem
Books	15% ad valorem
Butter	$2\frac{1}{2}$ ¢ a pound
Cigars and cigarettes	$4.50 a pound and 25% ad valorem
Cotton hose, pairs	$1.20 a dozen pairs and 30% to 50% ad valorem
Maple sugar	3¢ a pound
Olive oil in bottles	30¢ a gallon
Opera glasses	45% ad valorem
Pens	8¢ a gross
Silk wearing apparel	50% ad valorem

Exercise 78

1. Find the duty on 1000 gross of pens.

2. A traveler returning from Europe brings with him one automobile which cost $4700 and another that cost $7648. Find the amount of duty on them.

3. A returning traveler paid duty on the following: Opera glasses, costing $27.50; 2 dozen pairs of gloves, on which there was a duty of $2.25 a pair; $250 worth of silk wearing apparel; $65 worth of books. Find the total duty.

4. Find the duty on 400 dozen pint bottles of olive oil, costing 30¢ a pint; 300 pounds of maple sugar; and 2560 pounds of butter.

5. Find the duty on 4000 cigars costing $3 a hundred and weighing 15 pounds per thousand.

6. Find the duty on 4000 pairs of cotton hose invoiced at 8½¢ a pair, the ad valorem duty being 40%.

7. The sum of the invoice price and the duty on a purchase of opera glasses was $533.50. Find the invoice price.

8. The duty on an invoice of books was $126.90. Find the invoice price and total cost.

72. Income tax. Income taxes collected by the United States Government are of two kinds, **normal tax** and **surtax**.

Under the Revenue Act of 1918 the *normal tax* of an individual is computed for any year after 1918 by this

Rule. *Deduct from the net income $1000 in the case of an unmarried person and $2000 in the case of a married person. Deduct $200 for each dependent child. Of the remainder, take 4% of the first $4000, and 8% of the excess over $4000.*

A surtax is charged on all net incomes in excess of $5000. The rate of surtax is 1% on the first $1000 in excess of $5000, and increases 1% on each additional $2000 or fraction thereof.

Income tax rates are subject to change by Congress.

EXAMPLE 1. Find the income tax of a married man with two dependent children who has a net income of $7500. The rate of the surtax is 1% of the first $1000 in excess of $5000 and 2% of the next $2000.

SOLUTION. $2000 + $400 = $2400, the amount of deductions.
$7500 − $2400 = $5100, amount subject to normal.
4% of $4000 + 8% of $1100 = $248, amount of normal tax.
$7500 − $5000 = $2500, the amount subject to surtax.
1% of $1000 + 2% of $1500 = $10 + $30 = $40, amount of surtax.
$248 + $40 = $288, amount of income tax.

EXAMPLE 2. Find the income tax of an unmarried person who has a net income of $6000.

SOLUTION. $6000 − $1000 = $5000, the amount subject to normal tax.

4% of $4000 + 8% of $1000 = $160 + $80 = $240, the normal tax.

1% of $1000 = $10, the surtax.

$240 + $10 = $250, the total income tax.

Exercise 79

1. Find the income tax on a net income of $3500 of an unmarried man.

2. Find the income tax on a net income of $7500 of an unmarried man. The rate of surtax is 1% on the first $1000 in excess of $5000, and 2% on the remainder.

3. Find the income tax on a net income of $7000 of a married man, no dependent children. The rate of surtax is the same as in the preceding problem.

4. Find the income tax on a net income of $6800 of a married man with three dependent children. The rate of surtax is the same as in problem 2.

5. Find the income tax of a married man with no dependent children, who has a net income of $12,000. The rate of surtax is 1% on the first $1000 in excess of $5000, 2% on the next $2000, 3% on the next $2000, and 4% on the next $2000.

6. By the income tax law of 1918 the incomes of married persons with no dependent children were taxed as follows:

NET INCOME	TOTAL TAX	NET INCOME	TOTAL TAX
$ 3,000	$ 40	$ 100,000	$ 31,190
5,000	120	200,000	93,190
10,000	590	500,000	311,190
20,000	1990	1,000,000	663,190

Find the rate per cent of tax paid on each of these incomes correct to .01%.

73. Miscellaneous Government revenues. Shortly after the entrance of the United States into the European War internal revenues were placed on many articles that had not been taxed before, and other internal revenues were increased. Examples of these are given in the following exercises.

Exercise 80

1. A tax of 1¢ was levied on each 20¢ or fraction thereof which was paid for express, and 3% on freight charges. In one week a merchant paid $36.20 for freight and the following amounts for express packages: 40¢, 65¢, $1.12, 46¢, $2.35. Find the amount of tax that he paid on all of these.

2. A tax of 8% was levied on passenger fares above 35¢ and 10% on the amount paid for sleeping car berths. In one week a traveling man paid as railroad fare the following amounts: $1.45, 68¢, $1, $3.46, 28¢, 98¢, $1.60, $2.38, and $3.54. He paid for sleeping car tickets $1.50 and $2. Find the amount of taxes he paid.

3. A tax of 50¢ was levied on each real estate deed if the value exceeded $100 and was less than $500, with 50¢ additional for each additional $500 or fraction thereof. The tax on promissory notes was 2¢ on each $100 or fraction thereof. A farm of 240 acres was sold for $175 an acre. A payment of $26,000 was made in cash and the remainder by a promissory note. Find the tax on the deed and note.

4. At the end of a year a man attempts to compute as nearly as possible the amount of the war taxes that he has paid during the year. He has an income of $4500. He is married and has two dependent children. Find his income tax. He has bought an automobile of which the selling price is $1350. He has paid railroad fare to the amount of $126.42. He has bought 1200 cigars which are taxed at $4 a thousand. His express bills were $4, 68¢, and $2.34. Find the war taxes on these items. Find the total tax paid.

74. Insurance. A property owner may protect himself from loss by **insurance.** For a small per cent of the amount insured an **insurance company** will agree to pay the owner for his losses if the property is destroyed or damaged by certain agencies.

The contract between the insurance company and the person insured is called a **policy.**

The amount to be paid in case of loss is called the **face** of the policy.

The amount paid to the insurance company for the insurance is called the **premium.**

The premium is usually computed at a certain rate on each $100 insured for a given period. The periods most frequently used are 1 year, 3 years, and 5 years.

Three common forms of property insurance are **fire** and **lightning, tornado,** and **marine.**

By examining the records for a period of years an insurance company can determine with a high degree of accuracy the probable number of fires that will occur in a certain kind of location, as on farms, in villages, and in cities. From these facts the company can compute the premium to be charged to allow for losses and to secure profits. The probability of fire, and therefore the rate of insurance, depends upon the kind of building, as wooden or brick or stone, the fire protection, the nearness to other buildings, and the use of the building. If the number of fires increases, the insurance company will increase the rate of insurance. As losses must be paid from the premiums collected, it is to the advantage of every person insured to improve the protection from fire and to do everything possible to reduce the loss by fire.

In life insurance the insured pays an annual premium, in return for which the insurance company promises to pay to the insured or to someone he designates, a certain sum of money, either after a certain number of years or at the death of the insured.

Exercise 81

1. A dwelling house valued at \$6000 is insured for $\frac{4}{5}$ of its value for 5 years at \$1.50 a hundred. Find the premium.

2. Some farm buildings valued at \$3500 are insured against fire and lightning for $\frac{3}{5}$ of their value for 5 years at \$1.75 a hundred. Find the premium.

3. If the property in the last exercise is insured also against tornadoes the rate is \$2.50 a hundred. Find the premium in that case.

4. An automobile which cost \$800 was insured for $\frac{3}{4}$ of the cost the first year and for $\frac{5}{8}$ of the cost the second year at the rate of \$1 a hundred. Find the amount of the premiums for the two years. If a fire extinguisher had been carried, the rate would have been reduced 15%. How much would the premium have been reduced by carrying a fire extinguisher?

5. The rate on a city building is 90¢ a year. The rate for 3 years is twice the yearly rate, and for 5 years three times the yearly rate. How much is saved by taking out a policy for 5 years rather than by insuring annually if the building is valued at \$6000 and is insured for $\frac{4}{5}$ of its value?

6. The annual rate on a city building is 80¢. When a moving picture theater opens up next door the rate is increased 15%. What is the rate then? Can you give a reason for this increase in the rate?

7. The average annual rate for fire insurance in the United States is about 1% of the amount insured, and in Western Europe the rate is about .1%. The amount of fire insurance in the United States in a recent year, written by the leading companies, was \$50,000,000,000. How much would have been saved to policy holders if the rate of insurance had been as low as in Western Europe?

8. The estimated population of the United States in 1914 was 100,399,000. The estimated loss by fire was $182,836,000. What was the loss per capita to the nearest cent? The estimated loss per capita in Western Europe that year was 30¢. The loss per capita in the United States was how many per cent of that of Western Europe?

9. The estimated population of Chicago in a recent year was 2,450,000. The number of fires in that year was 12,447. The total loss of property was $6,018,589. What was the loss per capita? What was the average number of fires for each 1000 people?

10. In a certain city the annual rates on store buildings range from 80¢ on $100 to $1.50. The annual rate on a stock of dry goods is 25% higher, and on a stock of drugs 40% higher, than the rate on the building in which the stock is located. Find the range of the rates on each of these kinds of stocks.

11. In this city a merchant has a stock of dry goods worth $25,000 in a building valued at $8000. The rate on the building is 80¢ and the rate on the stock is 25% higher than on the building. The building is insured for 80% of its value and the stock for 75% of its value. Find the total premium.

12. What is the commission of the agent who wrote this insurance if he got 15% of the premium?

13. A man has decided to build a house on a certain plan. If he builds a frame house with a shingle roof, it will cost him $3600, and the rate of insurance will be $2.40 per $100 for 5 years. If he builds a brick veneered house with slate roof, it will cost him $4500, and the rate of insurance will be $1.80 per $100 for 5 years. If he builds the latter and insures it for $\frac{4}{5}$ of the cost, how much insurance will he save in 20 years?

CHAPTER X

BANKS AND BANKING

75. Promissory notes. When money is borrowed the borrower gives a **promissory note.** The following is a common form.

$ 2000.00 Denver, Colorado, August 16, 1920.

Three years after date I promise to pay to Arthur S. Porter or order

Two Thousand and 00/100 Dollars

for value received, with interest at the rate of six per cent per annum from date.

John R. Fremont

In this note John R. Fremont is the **maker**, Arthur S. Porter, the **payee**, and $2000, the **face** of the note. The day this note is due, August 16, 1923, is called the **date of maturity.**

76. To find the date of maturity of a note. To find the date of maturity of a note due in a certain number of days, count that number of days from the day the note is given.

EXAMPLE. Find the date of maturity of a note dated January 12, 1918, due in 90 days.

SOLUTION.

January,	19 days	The note is then due April 12.
February,	28 days	
March,	31 days	
April,	12 days	
Total	90 days	

If a note is due in a certain number of months, it is due on the same day of the month as that on which it was given, unless the month in which it is due has no corresponding day. For example, a note dated March 20, due in three months, falls due June 20 ; but a note dated March 31, due in 3 months, is due June 30. As a rule a note falling due on a holiday becomes due on the following day.

77. Indorsement of notes and checks. If a note or check reads " Pay to John Doe or order," " Pay to John Doe or bearer," or " Pay to the order of John Doe," it is said to be **negotiable**, that is, John Doe may transfer it to another person. What is said in the following discussion concerning notes is true also of checks.

If a note is payable to John Doe or bearer, it is payable to the person who holds it.

If a note is payable to John Doe or order, John Doe must, when transferring this note, **indorse** it, that is, he must write his name on the back of the note.

The usual forms of indorsement are **in blank** and **in full.** If the note given above is indorsed in blank, Arthur S. Porter writes his name on the back. If it is indorsed in full to James Smith, the indorsement is written : Pay to the order of James Smith, Arthur S. Porter.

A note indorsed in blank is payable to bearer. A note indorsed in full is payable only to the person to whom it is indorsed, unless this person also indorses it.

The indorser of a note becomes liable for its payment unless he writes the words *Without recourse* above his name. Thus in the indorsement given above Arthur S. Porter is liable for the payment of the note if John R. Fremont does not pay it. But if Arthur S. Porter writes the words " Without recourse " above his name in either the indorsement in blank or the indorsement in full, he is no longer liable for the payment of the note.

Forms of Indorsement

Indorsement in Blank	Indorsement in Full	Indorsement without Recourse
Arthur S. Porter	*Pay to the order of James Smith*	*Without recourse*
	Arthur S. Porter	*Arthur S. Porter*

Exercise 82

1. Write a note for $350, dated May 6 of the present year, due in six months, E. H. Cofer maker, and C. J. Crampton payee. Indorse this note in full to the Cary Clothing Company.

2. On March 25 Oliver Jones of Three Lakes, Wis., buys a team of horses from U. S. Walter for $365, and pays for them by a note due in one year with interest at six per cent. Write the note.

3. The note in the previous exercise is indorsed in blank and sold to James Maxwell, who indorses it in full to J. K. Crowley. Write these indorsements. Who now holds the note? When this note becomes due who should pay it? To whom? Name all the persons who are liable for the payment of this note.

4. Arthur S. Mills gives a note for $1500 to John Walsh, dated Aug. 10, due in 3 years, with interest at 7%. This note is sold to R. C. Mann to whom it is indorsed in full. Write the note and the indorsement. Who now holds this note? Who must pay it when due? To whom? Who gets the note when it is paid?

5. What is the date of maturity of a note dated Dec. 31 and due in 2 months? Of one dated Jan. 4 and due in 6 months?

78. Interest. The pupil has already learned how to compute interest. The following are two convenient forms of computation.

EXAMPLE 1. Find the interest on $900 at 7% from March 22, 1917, to May 11, 1919. Find time by subtracting dates.

SOLUTION. *Six per cent method.*

1919	5	11	Int. on $1 for 2 yr. 1 mo. 19 da. at 6% = $.128⅓.
1917	3	22	Int. on $900 for 2 yr. 1 mo. 19 da. at 6% = 900 × $.128⅓
2	1	19	= $115.35.

EXAMPLE 2. Find the interest on $1200 at 5% from June 17 to November 8. Use as the time the exact number of days.

SOLUTION. *Day method.*

Number of days in June	13	
July	31	
August	31	
September	30	
October	31	
November	8	
	144	

$$\frac{\$\cancel{1200}^{\cancel{12}} \times \cancel{5} \times \cancel{144}^{24}}{\cancel{100} \times \cancel{360}_{\cancel{30}_{6}}} = \$24.$$

Exercise 83

In the first five exercises find the time by subtracting dates. Find interest on

1. $3000 at 6% from Jan. 12, 1919, to March 15, 1921.
2. $654 at 5% from Sept. 15, 1918, to May 27, 1920.
3. $35.60 at 7% from July 3 to Dec. 31.
4. $700 at 5% from Feb. 27, 1920, to June 11, 1921.
5. $25,000 at 4% from July 1, 1919, to Jan. 1, 1923.

In the next four exercises find the exact number of days. Find the interest on

6. $800 from April 10 to Sept. 9 at 6%.
7. $1500 at 5% from Feb. 7, 1920, to Dec. 22, 1920.
8. $2460 at 4% from May 24 to Feb. 16.
9. $48.15 at 7% from Oct. 14, 1920, to May 5, 1921.

79. Depositing money. Most people leave their money at a bank for safe keeping. When money is deposited at a bank either the depositor or an officer of the bank fills out a **deposit slip.**

Deposited with
First National Bank
Dayton, Ohio

By .....
..... *19*

	Dollars	*Cents*
Currency		
Silver		
Gold		
Checks as follows:		
.....		
.....		
.....		

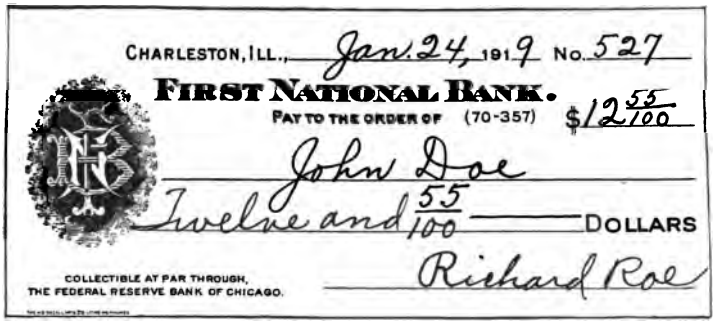
This deposit slip contains the name of the bank, the name of the depositor, the date of the deposit, the amount of the deposit, and the kind of funds deposited, whether currency, silver, gold, or checks.

When his first deposit is made the depositor receives a pass book in which is kept a record of the amounts of money deposited and withdrawn.

80. Checking account. Money that is deposited to be withdrawn in such amounts and at such times as the depositor wishes is kept in a checking account.

The depositor with a checking account may either withdraw the money himself, or order it paid to another person. In either case he fills out a check. The check contains (*a*) the name and the location of the bank on which it is drawn ; (*b*) the date when the check is made ; (*c*) the name of the person to whom the check is to be paid, who is called the **payee** ; (*d*) the amount to be paid, which is called the **face** of the check and is written both in figures and in words; (*e*) the signature of the **maker**, or **drawer**, of the check.

The name of the maker must be written by himself. The other parts of the check may be written by any one.


CHARLESTON, ILL., Jan. 24, 1919 No. 527

FIRST NATIONAL BANK.

PAY TO THE ORDER OF (70-357) $12 55/100

John Doe

Twelve and 55/100 DOLLARS

Richard Roe

COLLECTIBLE AT PAR THROUGH, THE FEDERAL RESERVE BANK OF CHICAGO.

When John Doe presents this check for payment he must indorse it. What has been said about the indorsement of a promissory note is true for the indorsement of a check.

When money is paid for a check the check is said to be **cashed.**

A check may pass through a number of hands before reaching the bank on which it is drawn.

When the First National Bank pays the above check it deducts $12.55 from Richard Roe's account. The check is then marked " Paid " and is later returned to Richard Roe.

Since John Doe has indorsed the check it becomes a receipt from him to Richard Roe for the payment of $12.55.

Many people pay all bills by checks. In this way they keep accurate accounts of all payments made, and the canceled checks become receipts.

If a depositor has drawn checks for an amount that is more than his deposits, he is said to have overdrawn his account, and the amount that he has overdrawn his account is called an **overdraft.** Most banks are prohibited by law from paying a depositor's checks when the depositor's account is overdrawn.

Banks usually desire to balance all pass books monthly. The balanced pass book shows the sum of the checks drawn, the sum of the deposits made, and the balance due the depositor.

Exercise 84

1. In the check on page 169 who is the payee? What is the face?

2. How may John Doe get the money for this check? Write the indorsement.

3. Give two reasons for paying bills by checks rather than by paying the money.

4. William Burke opens an account with the Merchants Bank of Indianapolis, Indiana, on January 3, 1920. He deposits $225 in currency, $46.50 in silver, and $238 in checks. Make out the deposit slip. What does Mr. Burke receive from the bank when he makes the deposit?

5. Three days later he buys a horse from Eugene Hays for $225, and pays for it with a check. Write the check.

6. How may Eugene Hays get the money for this check? Properly indorse the check.

7. On January 17 William Burke goes to the bank and draws out $40. Write the check that he must give the bank. Make it payable to Cash. Does such a check need indorsement?

8. During January and February William Burke makes the following deposits: January 25, $42; February 4, $195.46; February 16, $116. During these months he draws the following additional checks: January 25, $5.65; February 12, $10; February 18, $36.18; February 27, $176.30. On March 1 his pass book is balanced. What is his balance?

9. On March 17 William Burke sends his son John R. Burke, who is at college, a check for $100. Write this check. Tell the maker, face, and payee of this check.

10. Oscar Rodgers sells his chickens to John M. Wilkins. The chickens weigh 36 pounds and bring 18 cents a pound. Payment is made by a check on the Second National Bank of Peoria, Illinois. The check is dated July 24, 1921. Oscar

Rodgers indorses the check in full to M. N. Kopta, who indorses it in blank when he cashes it at a bank. Write the check and the indorsements.

11. A. M. Beals pays his month's bill of $5.37 due the Star Laundry with a check. The check is drawn on the Merchants Bank of St. Louis, Missouri, and dated June 2, 1920. Write the check. If this check is made payable to order, who must indorse it? Who finally gets the check? Should he keep it or destroy it? Give reasons for your answer.

12. Suppose that when the check mentioned in the last exercise is presented at the bank for payment, it is found that A. M. Beals has only $1.78 on deposit. What will probably happen?

13. R. M. Parker has a bank balance of $346.28. He draws the following checks : Aug. 10, $36 ; Aug. 18, $42.50 ; Aug. 27, $212.18 ; Sept. 6, $12.18. Find the condition of his bank account after these checks are cashed.

14. A note given by T. N. Flint, dated July 10, 1920, promises to pay H. R. Halifax $275 three months after date with interest at 7%. The note is paid by check when due. Write the check, which was drawn on the Citizens Bank of Chattanooga, Tennessee.

15. The check of problem 14 was cashed by A. R. Allison who deposited it in the First National Bank of Chattanooga. To whom will this check finally go? Mr. Allison indorses it in such a way that he does not become responsible for its payment. Write all the indorsements this check must finally have.

16. If a check payable to order were lost could it be collected by the finder if it had not the indorsement of the payee? Could it be collected by the finder if indorsed in blank by the payee? To whom would the check finally go? Would he know who had received the money?

17. Write a check which could be cashed by anyone holding it.

81. Borrowing money from a bank. As it is not probable that all of its depositors will want to withdraw their money at the same time, a bank finds it possible to lend a large part of its deposits and still have money enough to pay the checks of its depositors. It is the interest charged for the money lent that pays the bank for taking care of the deposits and for the necessary bookkeeping.

The following is a form of note used by many banks.

$ 500.00 No. 867 Due Sept 17

Chicago, Ill., July 19, 1914

Sixty days after date, for value received, We promise to pay to the order of

CONTINENTAL AND COMMERCIAL NATIONAL BANK OF CHICAGO, at its office,

Five Hundred and 70/100 DOLLARS,

with interest at the rate of seven per cent. per annum, after maturity until paid.

James R. Walker

Oscar B. Simons

Address:

The interest on $500 for 60 days at 7% is $5.83. This amount is deducted from the face of the note, and the remainder, $494.17, is the amount that the makers of the note receive from the bank.

The amount due on the above note at maturity is $500. The interest is computed on this amount and is collected when the loan is made.

Interest that is collected in advance, on the amount due at the maturity of the note, is called **bank discount.**

This note is due September 17. If it is not paid when due, it draws interest from maturity, September 17, to the date of payment.

The difference between the amount due at maturity and the bank discount is called the **proceeds.** In the above note the proceeds are $494.17.

The exact number of days from the day of discount to the day of maturity is called the **discount period.**

Exercise 85

Find the bank discounts on the following amounts:

1. \$800 for 90 days at 7%.
2. \$5000 for 6 months at 6%.
3. \$750 for 30 days at 5%.
4. \$1250 from May 10 to July 8 at 7%.
5. \$135 from Oct. 8 to Dec. 12 at 6%.
6. \$1000 from Jan. 10, 1919, to May 12, 1919, at 5%.
7. \$3000 from Dec. 1, 1919, to March 10, 1920, at 6%.
8. \$15,000 from Oct. 26 to Feb. 4 at 5%.
9. In the note on page 172 name the makers, payee, and face. What is the amount due at maturity? The discount period? The bank discount? The proceeds?
10. G. H. Banks gives a note for \$1200 to the Citizens Bank of Detroit, Michigan, for 90 days with interest at 7% from maturity. The date of the note is May 5, 1917. Write the note.
11. In the last exercise how much is due at maturity? What is the discount period? When is the note due? What is the bank discount? What are the proceeds? What amount does G. H. Banks get from the bank? What amount does he pay interest on?
12. Fill out the following table. Each note is discounted the day it is made.

Face	Date	Rate of Discount	Discount Period	Maturity	Bank Discount	Proceeds
\$800	Aug. 7	6%	90 days			
1800	May 9	6%	60 days			
450	Oct. 30	7%	30 days			
2200	Nov. 26	7%	10 days			

13. State the amount that the borrower receives from the bank in each part of the preceding exercise.

82. Selling non-interest-bearing notes to a bank. If James R. Roswell wishes to get the money for the following note before it is due, he may sell it to a bank. If it is sold the day it is made, the bank will discount it for 90 days at the rate of interest that it is charging for other loans and will pay James R. Roswell the proceeds.

800\frac{00}{100}$ Lebanon, Ohio, Sept. 21, 1919.

Ninety days after date I promise to pay to the order of James R. Roswell Eight Hundred and $\frac{00}{100}$ Dollars.

For value received.

William S. Montz

EXAMPLE. Find the discount and the proceeds on the above note if it is sold to a bank on November 20, and discounted at 7%.

SOLUTION. Ninety days after Sept. 21 is Dec. 20, the maturity of the note. From Nov. 20 to Dec. 20 is 30 days, the discount period.

$\frac{\$800 \times 7 \times 30}{100 \times 360} = \$4.67$, the bank discount.

$\$800 - \$4.67 = \$795.33$, the proceeds.

Exercise 86

1. Find the bank discount and the proceeds on the above note if it is discounted at 6% on the day it is made.

2. If the note is discounted as in exercise 1, how much does James R. Roswell receive from the bank? Who pays the note when due? To whom? What amount is paid? What amount does the bank make by the transaction? Who *gets* the note after it is paid?

3. A note for $625, John Doe maker, Richard Roe payee, without interest, dated Oct. 18, 1920, due in 9 months, is discounted at the State Bank Jan. 12 at 6%. Find the bank discount and the proceeds.

4. The following note was sold to the Citizens Bank of Greensboro, March 30. The bank discounted the note at 7%. Find the bank discount and the proceeds.

$1500.00

Greensboro, North Carolina, Jan. 17, 1918.

Ninety days after date I promise to pay to the order of M. A. Kizer

Fifteen hundred and 00/100 Dollars.

Value received.

E. M. Rose

5. What is the face of the above note? Who is the maker? Who is the payee? Who gets the note March 30? From whom? Who must indorse the note when it first changes ownership? What sum does the bank pay for the note? Who receives the proceeds? When? Who receives the bank discount? When? What is the date of maturity? Who pays the note at maturity? How much is paid? Who receives the note after it is paid at maturity?

6. At a farm sale W. R. Baker buys of John Price a horse for $168 and some hogs for $58.40, and gives a note for the amount. The note is dated March 19, 1919, and is due in 6 months without interest. On March 25 this note is discounted by the payee at the Second National Bank at 7%. How much is received for the note?

7. Answer questions similar to those in exercise 5 concerning the note in exercise 6.

8. Copy and fill out the following table:

Face	Date	Time to Run	Rate of Discount	Date of Discount	Maturity	Discount Period	Bank Discount	Proceeds
$225	March 5	90 da.	6%	April 17				
48.75	May 31	4 mo.	6%	July 12				
326	June 17	60 da.	7%	July 10				
3400	Sept. 24	3 mo.	7%	Nov. 10				

9. A wholesale house sells goods for $354.50 on 60 days' time and takes in payment a note due in 60 days. The note is discounted 2 days later at 6%. What are the proceeds?

10. I sell A a bill of goods for $1000 and allow him a discount of 3% for cash. I sell B a bill of goods for the same amount and take in payment a note due in 6 months without interest. The same day I discount the note at a bank at 7%. Which method is better for me and by how much?

11. M. R. Bond borrows money from a bank. He gives the bank a note for $850 due in 30 days, the note is discounted at 6%, and he receives the proceeds. How much does he receive from the bank? How much does he pay interest on? What interest does he pay? What rate of interest does he pay on the money that he gets from the bank?

12. If I give a note due in 2 months for $1750 to a bank and pay the interest at 6% in advance, how much do I receive? How much must I pay when the note is due?

13. Some banks pay 3% interest per annum on money that is deposited for 6 months or more. At this rate what is the interest on $1000 for one year? If the bank lends this money at 7%, how much profit does the bank make?

14. If I give to a bank a note for $1000 due in 1 year, and pay 7% interest in advance, how much money do I get from the bank? How much do I pay for the use of this money? What rate of interest is that on the money that I get?

83. Selling interest-bearing notes. If this note were sold to the First National Bank of Atchison on August 27,

$1200.00 Atchison, Kansas, May 10, 1918.

One year after date I promise to pay to the order of H. B. Ashton

Twelve Hundred and 00/100 Dollars with interest at seven per cent per annum from date, value received.

Milton J. Wiley

and the bank discounted it at 7%, the proceeds would probably be found in the following manner :

The interest on $1200 for one year at 7% is $84. The amount of the note at maturity is $1284. The date of maturity is May 10, 1919. The exact number of days from August 27, 1918, to May 10, 1919, is 256 days. The simple interest on $1284 at 7% for 256 days is $63.91. This is the bank discount. The bank discount subtracted from the amount due at maturity leaves $1220.09, the proceeds to be paid to the person selling the note.

Rule for finding the bank discount on an interest-bearing note. *Find the amount of the note due at maturity. Find the simple interest on this amount for the exact number of days from the date of discount to maturity, at the rate of discount used. This is the bank discount.*

NOTE. The above rule is not always followed by banks. There seems to be no rule that is always followed. The above rule is probably in more general use than any other. The amount that a bank is willing to pay for an interest-bearing note depends upon who is selling the note, the security of the note, and the amount of money that the bank has on hand to lend.

From some investigation which the authors have made concerning the practice of banks in discounting interest-bearing notes it appears that large city banks use the above rule, but that the practices of small banks differ widely.

We have now studied three cases of bank discount: borrowing money from a bank; selling a non-interest-bearing note; selling an interest-bearing note. In each case *the bank discount is found as the simple interest at the rate of discount on the amount of the note due at maturity for the exact number of days from the day of discount to maturity.*

Exercise 87

1. What is the face of the note on page 177? Who is the maker? Who is the payee? Who gets the note August 27? From whom? Who must indorse the note when it first changes ownership? What is the discount period? On what amount is the bank discount reckoned? What is the bank discount? What sum does the bank pay for the note? Who receives the proceeds? When? Who receives the bank discount? When? When does the note mature? Who pays the note at maturity? How much is paid? Who receives the note after it is paid at maturity?

2. Find the bank discount and the proceeds on the following note discounted Dec. 30 at 7%.

$ 2500.00 *Detroit, Mich., Oct. 19, 1918.*

Six months after date I promise to pay to the order of James B. Highsmith

Two Thousand Five Hundred and $^{00}/_{100}$ *Dollars for value received with interest at six per cent per annum from date.*

George R. Wakefield

3. Answer the questions in exercise 1 concerning the note in exercise 2.

4. Copy and fill out the following table:

Face	Date	Time to Run	Rate of Int.	Date of Dis.	Rate of Dis.	Maturity	Dis. Period	Bank Dis.	Proceeds
$ 500	May 6	1 yr.	6%	July 31	6%				
964	July 9	30 da.	5%	Aug. 1	6%				
300	Oct. 21	90 da.	7%	Dec. 16	7%				
3500	Sept. 1	3 mo.	4%	Nov. 30	6%				

5. On June 1 Robert Connell buys a threshing machine from a dealer for $2400 and gives in payment three notes without interest. One note is for $1000 and is due in 6 months ; one is for $600, due in 1 year ; and the third is for $800 and is due in 18 months. The dealer sells these notes to a bank August 22. Find the proceeds, if discounted at 5%.

Note. When the discount period is more than one year count the number of years and the exact number of days in the remaining part of a year.

6. A man sells a house and lot for $3200. He takes in payment $1500 cash and a note for the balance due in one year and bearing 5% interest. He at once discounts the note at a bank at 6%. How much cash does he receive for his property?

7. A hardware dealer sells farming implements which cost him $3456 at a profit of 25%. He takes in payment notes due in 90 days without interest. He discounts $1200 of these notes at a bank the day they are given at 7%. He discounts $1500 of them 45 days after they are given at 7%. One note for $200 is paid in full when due. The remainder of the notes are not paid promptly and the dealer pays an attorney 5% for collecting them. What were the dealer's net profits on the whole transaction?

84. Compound interest. A principal is said to be drawing **compound interest** if at the end of each period when interest is due the interest is added to the principal and the interest for the next period is computed on this sum.

If the interest is added to the principal at the end of each year, the interest is said to be **compounded annually**; if the interest is added to the principal at the end of each six months, the interest is said to be compounded **semi-annually**; if at the end of each three months, the interest is said to be compounded **quarterly.**

EXAMPLE 1. Find the compound interest on $1000 at 5% for 2 years compounded annually.

SOLUTION.

$1000	
.05	
50.00	interest for the first year.
1000	
1050	amount at end of the first year.
.05	
52.50	interest for the second year.
1050	
1102.50	amount at end of the second year.
1000	
$102.50	compound interest for two years.

EXAMPLE 2. John deposits $100 in a savings bank which pays 4% interest compounded semi-annually. How much is due him at the end of a year?

SOLUTION.

$100	
.02	
2	interest for the first 6 months.
100	
102	amount due at the end of the first 6 months.
.02	
2.04	interest for the second 6 months.
102	
$104.04	amount due at the end of the year.

Exercise 88

Find the interest compounded annually on

1. \$1000 for 2 years at 4%.
2. \$100 for 3 years at 3%.
3. \$800 for 4 years at 2%.
4. \$375 for 5 years at 4%.
5. \$450 for 3 years at 2%.
6. \$86.40 for 2 years at 4%.

Find the interest compounded semi-annually on

7. \$600 for 2 years at 3%.
8. \$1800 for 1 yr. 6 mo. at 2%.
9. \$648 for 3 years at 4%.

Find the interest, compounded quarterly, on

10. \$900 for 1 year at 4%.
11. \$80 for 9 months at 3%.
12. \$235 for 1 year at 5%.
13. \$600 for 6 mo. at 4%.

14. Find the interest compounded annually on \$600 for 3 yr. 8 mo. 10 da. at 4%.

SOLUTION. The compound amount of \$600 for 3 yr. at 4% = \$674.92.

The interest on \$674.92 for 8 mo. 10 da. at 4% = \$18.75.

\$674.92 + \$18.75 − \$600 = \$93.67, the compound interest.

Find the interest compounded annually on

15. \$1000 for 2 yr. 6 mo. at 4%.
16. \$3500 for 2 yr. 5 mo. 12 da. at 2%.

85. Compound interest by tables. The laws of most states prohibit the collection of compound interest on notes. But compound interest is of much importance in finding the amount due on deposits in savings banks, and in finding the returns on long-time investments such as are made by insurance companies, banks, and other corporations that expect to collect the interest when due and reinvest it. In such cases compound interest is computed by the use of tables.

A Table Giving the Compound Amount of $1 for Any Number of Periods up to 20.

Periods	1 per Cent	1½ per Cent	2 per Cent	2½ per Cent	3 per Cent	Periods
1	1.010000	1.015000	1.020000	1.025000	1.030000	1
2	1.020100	1.030225	1.040400	1.050625	1.060900	2
3	1.030301	1.045678	1.061208	1.076891	1.092727	3
4	1.040604	1.061364	1.082432	1.103813	1.125509	4
5	1.051010	1.077284	1.104081	1.131408	1.159274	5
6	1.061520	1.093443	1.126162	1.159693	1.194052	6
7	1.072135	1.109845	1.148686	1.188686	1.229874	7
8	1.082857	1.126493	1.171660	1.218403	1.266770	8
9	1.093685	1.143390	1.195093	1.248863	1.304773	9
10	1.104622	1.160541	1.218994	1.280085	1.343916	10
11	1.115668	1.177949	1.243374	1.312087	1.384234	11
12	1.126825	1.195618	1.268242	1.344889	1.425761	12
13	1.138093	1.213552	1.293607	1.378511	1.468534	13
14	1.149474	1.231756	1.319479	1.412974	1.512590	14
15	1.160969	1.250232	1.345868	1.448298	1.557967	15
16	1.172579	1.268985	1.372786	1.484506	1.604706	16
17	1.184304	1.288020	1.400241	1.521618	1.652847	17
18	1.196147	1.307341	1.428246	1.559659	1.702433	18
19	1.208109	1.326951	1.456811	1.598650	1.753506	19
20	1.220190	1.346855	1.485947	1.638616	1.806111	20

Periods	3½ per Cent	4 per Cent	4½ per Cent	5 per Cent	6 per Cent	Periods
1	1.035000	1.040000	1.045000	1.050000	1.060000	1
2	1.071225	1.081600	1.092025	1.102500	1.123600	2
3	1.108718	1.124864	1.141166	1.157625	1.191016	3
4	1.147523	1.169859	1.192519	1.215506	1.262477	4
5	1.187686	1.216653	1.246182	1.276282	1.338226	5
6	1.229255	1.265319	1.302260	1.340096	1.418519	6
7	1.272279	1.315932	1.360862	1.407100	1.503630	7
8	1.316809	1.368569	1.422100	1.477455	1.593848	8
9	1.362897	1.423312	1.486095	1.551328	1.689479	9
10	1.410598	1.480244	1.552969	1.628895	1.790848	10
11	1.459970	1.539454	1.622853	1.710339	1.898299	11
12	1.511069	1.601032	1.695881	1.795856	2.012197	12
13	1.563956	1.665074	1.772196	1.885649	2.132928	13
14	1.618695	1.731676	1.851945	1.979931	2.260904	14
15	1.675349	1.800944	1.935282	2.078928	2.396558	15
16	1.733986	1.872981	2.022370	2.182875	2.540352	16
17	1.794676	1.947901	2.113376	2.292018	2.692773	17
18	1.857489	2.025817	2.208478	2.406619	2.854339	18
19	1.922501	2.106849	2.307860	2.526950	3.025600	19
20	1.989789	2.191123	2.411714	2.653298	3.207136	20

EXAMPLE 1. Find the amount due and the compound interest on \$600 for 10 years at 4%, compounded annually.

SOLUTION. From the table the compound amount of \$1 for 10 years at 4% is \$1.480244. Then the amount of \$600 is $600 \times \$1.480244 = \$888.15$. Subtracting \$600 from this amount leaves \$288.15, the compound interest.

EXAMPLE 2. Find the amount due and the compound interest on \$560 for 3 years at 4%, compounded quarterly.

SOLUTION. Three years equal 12 quarters. One per cent interest is due at the end of each quarter. Hence the problem is the same as that of finding the compound interest for 12 years at 1%. From the table the compound amount of \$1 for 12 years at 1% is \$1.126825. $560 \times \$1.126825 = \$631.02$, the compound amount. The compound interest is $\$631.02 - \$560 = \$71.02$.

Exercise 89

Use the table and find the compound interest on

1. \$1200 for 6 years at 2%, compounded annually.

2. \$859 for 10 years at 4%, compounded annually.

3. Find the interest in exercise 2, compounded semi-annually. What is the difference in the two results?

4. \$265.80 for 3 years at 4%, compounded quarterly.

5. \$10 for 5 years at 6%, compounded quarterly.

6. \$482 for 8 years at 3%, compounded semi-annually.

7. \$1,000,000 for 10 years at 4%, compounded semi-annually.

8. Mr. Tucker has a loan fund of \$5000 which he lends at 6%. At the end of each year he lends all the interest which he has collected for the preceding year. What will the fund amount to at the end of 2 years? At the end of 3 years? Of 4 years?

9. George places $500 in a savings bank which pays 3% interest, compounded annually. How much is due him at the end of the first year? At the end of the second year? At the end of the fourth year?

10. Which is the better, to deposit $1000 in a savings bank which pays 3½%, compounded annually, or in a savings bank which pays 3%, compounded quarterly, if the money is to be left there 2 years?

11. How much more interest will a depositor get on $400 deposited in a savings bank which pays 4%, compounded quarterly, than in a savings bank which pays 4%, compounded semi-annually, if the money is left on deposit for 18 months?

12. A newsboy deposits in the savings bank $45 and at the end of every 3 months makes a deposit of $15. The bank pays 4% interest, compounded quarterly. How much does it owe him at the end of 1 year, 9 months?

13. John White pays a life insurance premium of $264 each year. If the insurance company gets 3½% interest, compounded annually, on these amounts, how much has it received from his payments at the end of 5 years?

14. A bank lends $25,000 in 1 day. It charges 7% interest and makes the loans for 1 year. How much does the bank make by deducting the interest in advance and lending this interest at 7% for the year?

86. Savings banks. The depositor in a savings bank receives a pass book as does a depositor with a checking account. The money in the savings account can be drawn out only by the depositor on presentation of the pass book, and cannot be drawn out by check.

Savings banks usually pay interest compounded semi-annually. As a rule the interest is added to the principal on January 1 and July 1. The following illustrates one method of computing the interest on a savings account.

Suppose that the customer has a deposit of $300 on January 1 and that the bank pays 4% interest, compounded semi-annually. Suppose that the interest is added to the principal on January 1 and July 1. If the depositor makes neither a deposit nor a withdrawal before July 1, there is added to his account on July 1 the interest on $300 for 6 months at 4%, which is $6.

If, however, on February 10 he makes a deposit of $60 and on May 1 a deposit of $40, there is added to his account on July 1 the interest on $300 for 6 months, the interest on $60 from March 1 to July 1, and the interest on $40 from May 1 to July 1. That is, the depositor receives interest on a new deposit from the date it is made if it is made on the first of a month, as on May 1, and from the first day of the following month if the deposit is not made on the first of a month, as on February 10.

If a depositor who has a deposit of $300 on January 1 should withdraw $50 on any day between January 1 and July 1, he would receive interest on $250 from January 1 to July 1.

87. Postal savings banks. The United States Government operates savings banks in connection with the post office department. Any person of the age of 10 years or over may become a depositor. An account cannot be opened for less than $1, nor can fractions of $1 be deposited or withdrawn. No person is permitted to deposit more than $100 in any one calendar month, nor to have a balance to his credit at one time of more than $500, exclusive of interest.

Interest is allowed at the rate of 2% for each year that the money remains on deposit, beginning on the first day of the month following the month in which the deposit is made. Interest may be collected at the end of each year but is not compounded. No interest is allowed for periods less than one year.

88. War-Savings Certificate Stamps. In the year 1918 the United States Government began to sell **Thrift Stamps** and **War-Savings Certificate Stamps.**

A Thrift Stamp costs 25¢. When a sufficient number of Thrift Stamps are collected they may be exchanged for **War-Savings Certificate Stamps.**

A War-Savings Certificate Stamp of the series of 1918 costs $4.12 in January, 1918, and increases in value 1¢ each month up to and including December, 1922. It is due January 1, 1923, and is then worth $5. This increase amounts to 4% interest, compounded quarterly, if the stamp is held until due. These stamps may be cashed at full value at any time at any post office after 10 days' notice. The cost of a War-Savings Certificate Stamp of the series of 1918 increases 1¢ a month after January 1, 1918, up to and including December 31, 1918.

Exercise 90. Problems about Savings

1. On January 1 Robert has a deposit of $80 in a savings bank. On May 10 he withdraws $10. The bank pays 4% interest, compounded semi-annually. How much is due Robert July 1? On August 1 he deposits $25, and on October 27 he deposits $10. How much is due him January 1?

2. On January 1 my balance in the savings bank is $212. During the year I make the following deposits: March 1, $10; June 24, $45. I also make the following withdrawals: February 5, $12; August 5, $10. What is due me on January 1, the following year? The bank pays 3% interest, compounded semi-annually.

3. A newsboy deposits in a savings bank $5 each month at the end of the month for 1 year beginning in January. What is due him at the end of the year? The bank pays 3% interest, compounded semi-annually.

4. If $50 is deposited in a postal savings bank and left there for 2 years, how much interest is earned?

5. The following deposits were made in a postal savings bank: Feb. 1, 1917, $25; May 14, 1917, $12; Aug. 4, 1917, $30; Jan. 26, 1918, $40; May 6, 1918, $25. Find the amount of interest that has been earned up to Jan. 1, 1920. Remember that interest is paid only for whole years that the money is left on deposit.

6. A boy buys 3 War-Savings Certificate Stamps in 1918, 1 in February, 1 in July, and 1 in September. What is the total cost? If he keeps them until January 1, 1923, how much will he receive for them? That is how much more than the cost?

7. Show that an investment of $4.12 in a War-Savings Certificate Stamp on January 31, 1918, yields approximately 4% interest, compounded quarterly, if $5 is received for the stamp on January 1, 1923. That is, that $4.12 put at interest compounded quarterly, at 4% amounts to $5, approximately, in 4 yr. 11 mo.

8. Find the difference between the simple interest on $100 for 12 years at 5% and the compound interest, compounded annually, at 4% on the same amount for the same time.

9. A boy gets a birthday present of $100 when he is 12 years old. If it is placed in a savings bank where it draws interest at 4%, compounded semi-annually, how much will be due the boy when he is 21 years old?

10. A man hides $750 in his home for 18 years. If he had deposited this money for this time in a savings bank which pays 4% interest, compounded annually, how much would have been due him at the end of 18 years?

11. Is there a savings bank in your community? What rate of interest does it pay? When are its interest paying times? What is the smallest sum that may be deposited?

89. Partial payments on a note. When money is borrowed from banks, loan companies or others in the loan business, it is not customary to permit the principal of a loan to be paid before it is due, unless the interest is paid for the full time of the loan. In some notes it is stated that a part or all of the principal may be paid at any interest paying time. But in many cases of small loans between individuals the borrower is permitted to make part payment on a note at any time he has the money.

A payment made on a note before it is due is called a **partial payment.**

In such cases the United States Supreme Court has ruled that,

Partial payments on notes must first be used to cancel the interest due. Any balance remaining may be used to lessen the principal. If, however, the payment is too small to pay the interest due, the unpaid interest must not be used to increase the principal, which must never represent more than the money actually and previously due.

This is equivalent to the following

Rule. *Find the amount of the note from the date of the note until the date of the first payment. If this payment is more than the interest then due, deduct the payment from the amount. Use the remainder as a new principal and compute the amount to the date of the next payment. Deduct this payment and proceed as before. In case any payment is less than the interest due at the time the payment is made, compute the amount of the note to the date when the sum of the payments exceeds the interest due.*

EXAMPLE. A note dated January 25, 1919, principal $3275, bearing interest at 6%, has the following indorsements showing payments made : Dec. 15, 1919, $525 ; Oct. 20, 1920, $100 ; Jan. 24, 1921, $750. How much is required to pay the note in full July 1, 1921?

SOLUTION. The time is found from the date of the note until the time of the first payment, then from the time of each payment to that of the next.

The interest on \$1 at 6% for this time is placed after the time.

The amount of each payment is placed after its date.

The interest on \$3275 from the date of the note until the date of the first payment is $3275 \times \$.053\frac{1}{3} = \$174.67$. The first amount = \$3449.67.

Deducting the first payment gives the second principal, \$2924.67. The interest on \$2924.67 from the time of the first payment to the time of the second payment is \$148.67.

Since the second payment is less than the interest due, the second payment is added to the third payment, and the interest on \$2924.67 until the third payment is found to be \$194.49.

The amount of \$2924.67 until the third payment is \$3119.16.

The third principal after deducting the sum of the payments is \$2269.16.

The amount of \$2269.16 until the date of settlement is \$2328.54.

1919	12	15	\$525	3275	
1919	1	25		$.053\frac{1}{3}$	194.49
	10	20	.05	1092	2924.67
			$.003\frac{1}{3}$	9825	3119.16
			$.053\frac{1}{3}$	16375	850.
				174.667	2269.16
1920	10	20	\$100	3275.	$.026\frac{1}{6}$
1919	12	15		3449.67	37819
	10	5	.05	525.	1361496
			$.000\frac{5}{6}$	2924.67	453832
			$.050\frac{5}{6}$	$.050\frac{5}{6}$	59.37635
1921	1	24	\$750	6)1462335	2269.16
1920	10	20		243723	2328.54
	3	4	.015	1462335	
			$.000\frac{2}{3}$	148.67073	
			$.015\frac{2}{3}$	2924.67	
			$.050\frac{5}{6}$	$.066\frac{1}{2}$	
			$.066\frac{1}{2}$	146234	
1921	7	1		1754802	
1920	1	24	.025	1754802	
	5	7	$.001\frac{1}{6}$	194.49056	
			$.026\frac{1}{6}$		

Exercise 91

1. A note for $325 was dated June 16, 1924. The rate of interest was 6%. A payment of $130 was made on May 4, 1925. How much was due at the time of settlement, August 30, 1925?

2. A note for $430 with interest at 6% was dated May 15, 1915. A payment on it was made May 15, 1916, of $125. A second payment of $85 was made Nov. 15, 1917. How much was due on May 15, 1918?

3. A note for $2790 bearing interest at 7%, dated July 4, 1920, has payments indorsed as follows : April 20, 1921, $430; Aug. 16, 1921, $215 ; Dec. 3, 1922, $580. Find the amount due July 4, 1923.

4. A note for $850 payable in 3 years with interest at 5%, payable semi-annually, has an indorsement of $95 paid at the first interest paying time ; $170 paid at the second interest paying time ; and $235 paid at the fourth interest paying time. At the other interest paying times only the interest was paid. How much was due at maturity?

CHAPTER XI

STOCKS AND BONDS

90. Investment in a partnership.

1. A and B go into partnership and buy a dry-goods store. A invests $3000 and B invests $2000. They hire C to take charge of the store and agree to share the profits according to the amount of money invested. What part of the capital has each invested? What part of the profits should each receive?

2. The first year the net profits were $800. How much did each partner receive? The profits were what per cent of the money invested?

3. For the first three years the net profits each year averaged 15% of the money invested. What were the total net profits for the first three years? What were A's profits? What were B's profits?

4. If the same rate of profits continues, is this store a good investment? How does it compare with lending money in your community, with good security, at the rate usually paid?

5. If money can be lent for 6%, how much must be lent to secure an income equal to the average yearly income from this store? How much, if money is lent at 5%? How much in a Postal Savings Bank at 2%? How much must be invested in Liberty Bonds at $4\frac{1}{4}$%?

6. If a man wants to invest so as to secure 6% on his investment, how much can he afford to pay for this dry-goods store if the net profits continue to average annually 15% of the capital that A and B invested? How much can he afford to pay if he wants to make 8% on his investment?

91. Organizing a corporation. A and B wish to enlarge their dry-goods business. To do this they must have more money. There are other men who are willing to invest money in the store if A and B will organize a corporation or stock company, but who are not willing to invest in a partnership. We shall see some reasons why investment in a stock company may be preferred to investment in a partnership.

The money contributed by the members of the corporation to carry on the business is called the **capital.**

The capital is divided into **shares.** The shares are called **stock**, and the persons who own the stock are called **stockholders.** A share is usually $100, but may be more or less. The amount of capital that a share represents is called its **par value** or **face value.**

A and B get permission from their state government to organize a corporation for the purpose of carrying on a mercantile business. Each takes a certain amount of stock and they find other men who are willing to take the remainder. Persons who agree to take stock in a corporation, that is, agree to invest money in it, are said to **subscribe** the stock.

After all the stock is subscribed the stockholders meet and organize, electing a president, secretary, treasurer, and other necessary officers, and a board of directors. A is elected president and B is elected treasurer. The state now issues to these stockholders a **charter** which defines the rights and powers of the corporation. The name of the corporation is The Oswego Mercantile Company. On the following page is one of its stock certificates.

In speaking of the profits in the following exercises we shall mean the **net profits**, that is, the profits left after all expenses are paid. When the net profits are distributed among the stockholders they are called **dividends.** *Dividends are always computed at a certain per cent of the par value of the capital stock.*

CAPITAL STOCK	SHARES
$20,000	$100 each
No. 6	20 Shares

This Certifies that R. S. Miller is the owner of Twenty shares of the Capital Stock of *The Oswego Mercantile Company*, transferable only on the books of the Corporation by the holder hereof or by Attorney upon surrender of this Certificate properly indorsed.

In witness whereof the said Corporation has caused the certificate to be signed by its duly authorized officers, and to be sealed with the Seal of the Corporation at Oswego, New York, this twelfth day of March, 1920.

WILLIAM H. SCOVILLE, *President* (Seal) ARTHUR H. WILLIAMS, *Treasurer*

Each stockholder gets a **stock certificate**, which tells the par value of a share and the number of shares he owns.

Many corporations issue two kinds of stock, **preferred** and **common**. As a rule preferred stock receives a fixed rate of dividend if so much is earned, and the remaining dividend goes to common stock. However, methods of division of dividends between preferred and common stock differ widely.

92. Advantages of a stock company over a partnership.

(*a*) Each member of a partnership is liable for all of its debts. If a partnership fails, a member may lose not only all that he has invested in the partnership, but other property not invested in the partnership may be taken to pay its debts.

If a stock company fails, a stockholder loses no more than he has invested except in certain cases specified by law.

(*b*) One partner may not withdraw from a partnership without the consent of the other partners.

A stockholder may sell his stock at any time.

(*c*) The withdrawal of a partner dissolves a partnership.

The withdrawal of a stockholder does not dissolve a corporation.

Exercise 92

1. What is meant by the capital of a corporation?

2. A share usually represents how much capital?

3. What is meant by the par value of a share? By the face value?

4. Can you name some corporations doing business in your county?

5. State some advantages of a corporation over a partnership.

6. If X rents a house from Y for $30 a month, what is the income that Y receives called? If money is lent, what is the income from it called? What is the income from stocks called?

7. The income from stocks is reckoned as a per cent of what?

8. The Oswego Mercantile Company was organized with a capital stock of $20,000. A and B exchanged their dry-goods business for $10,000 of this stock, of which A got $\frac{3}{5}$ and B $\frac{2}{5}$. C, D, E, and F each took $2000 worth of stock, and G and H each took $1000. The stock was divided into shares of $100 each. How many shares did each stockholder receive?

9. At the end of the first year the stockholders received a 4% dividend. How much was that per share? How much dividend did each stockholder receive?

10. The average rate of the dividends for the first three years was 9%. How much income did A receive in the first three years? How much would he have received if he had lent his money at 6%? How much more did he gain by investing in the corporation? How much money must he have lent at 6% to receive as much income as he received from his stock? Was this stock a good investment? Can you name two things that must be true about an investment to make it a good investment?

11. With the dividends at 9% how much does C receive annually from his stock? That is 6% on how much money? Suppose that C offers to sell you his stock. How much can you afford to pay him so that you will receive 6% on the money invested, if the dividends continue the same?

12. Suppose that you pay C $120 a share for his stock. How much must you pay for all of C's stock at that rate?

13. The first year that you own the stock it pays a 6% dividend. How much is that a share? What per cent is that on the money paid for the stock? How much would you have received if you had lent the money at 6%? Which is better and by how much?

14. How much dividend must each share yield to pay you 8% on the money invested?

15. Suppose that during the second and third years that you own the stock, business is poor and The Oswego Mercantile Company pays only 3% dividends. Can you probably sell your stock for as much as you paid for it? What income would you get if your money were deposited in a savings bank at 3%? Would that be better than owning the stock?

16. What rate of dividend is paid when B receives $260 dividends annually?

17. If the net profits of this company are $1546.68 and a 7% dividend is paid, what surplus remains?

18. If The Oswego Mercantile Company pays an average annual dividend of 6%, how many shares of it are needed to bring as much income as $1200 worth of building and loan stock which pays 7% annually?

19. If the net profits of this company become small, say 2% a year, what effect will that have upon the selling price of the stock?

20. If it is thought that the company is likely to become bankrupt, what effect is that likely to have upon the selling price of the stock?

93. Bonds. When a corporation borrows money, it issues bonds. A **bond** is a promise of a corporation to pay a certain sum of money at a certain time with a certain rate of interest. A bond is simply the promissory note of a corporation.

Bonds are issued by various departments of the government, as the national government, the government of a state, a county, or a school district.

Bonds are usually secured by a mortgage on the property of the corporation.

The interest on bonds is paid annually, semi-annually, or quarterly, according to agreement.

The market value of bonds, like that of other property, rises and falls. The market value of bonds depends upon the rate of interest and the financial security of the corporation.

The name and address of the owner of a **registered** bond are registered at the offices of the corporation. When interest on a registered bond is due, it is sent directly to the owner.

A **coupon** bond has attached to it interest coupons, which are promises to pay interest. When interest is due, the owner of the bond detaches the coupon and presents it for payment.

94. Differences between stocks and bonds. Stocks represent money invested in the corporation. Bonds represent money lent to the corporation. The stockholders own the property of the corporation. The bondholders lend money to the corporation.

The income from stocks is **dividends**. Dividends are computed as a per cent of the par value of the stock. The amount of dividend varies with the amount of net profits. If there are no net profits, there are no dividends.

The income from bonds is **interest**. The rate of interest is fixed in the bond, and is a certain per cent of the face of the bond.

Interest on bonds must be paid before dividends on stocks.

95. Buying and selling stocks and bonds. We have seen that stock represents money invested in a corporation, and that bonds are like promissory notes.

Like other property stocks and bonds are bought and sold. If stocks or bonds sell for more than par value, they are said to be **above par.** If they sell for less than par value, they are said to be **below par.** If they sell for par value they are said to be **at par.**

Thus, if a share of The Oswego Mercantile Company stock sells for \$112, it is \$12 above par ; if it sells for \$94, it is \$6 below par ; if it sells for \$100, it is at par.

Just as there are persons who make a business of buying and selling dry-goods, or groceries, or grain, so there are persons who make a business of buying and selling stocks and bonds. A person whose business is the buying and selling of stocks and bonds is called a **stock broker.**

Stock brokers usually do their buying and selling in a stock exchange. Stock exchanges are found in certain large cities. The principal stock exchange in the United States is in New York City.

A broker usually charges $\frac{1}{8}\%$ of the par value for buying and selling stocks and bonds. This is called **brokerage.**

The pupil should remember that the *brokerage is always a per cent of the par value* of the stocks or bonds that are bought or sold. If, for example, a broker receives $\frac{1}{8}\%$ brokerage for buying 10 shares at \$80 each, and also for buying 10 shares at \$127 each, par value in each case being \$100, the brokerage is \$1.25 in each case.

The price for which stock sells is called its **market value.**

A report of the sales and the market prices of stocks and bonds may be found in the daily papers. Some quotations are given on the next page. Get a daily paper of some large city and find the stock quotations. These usually give the price offered, or bid, for stock, also the price asked by those wishing to sell stock.

STOCK QUOTATIONS

American Express	131¾	United States Steel, common	106¼
Illinois Central	105⅞	United States Steel, preferred	119½
Missouri Pacific	15¼	Standard Oil of Indiana	810
Pullman Car Company	163	Bethlehem Steel, common	525
Massachusetts Electric Railway	4¾	Bethlehem Steel, preferred	135

BOND QUOTATIONS

United States 2s, registered	99¾	Anglo-French 5s	93¼
United States 2s, coupon	99¾	Chicago and Northwestern 3½s	84½
United States 4s, registered	110	Lake Shore 4s	95⅝
United States 4s, coupon	110½	Rock Island 4s	76

The quotation, "American Express 131¾," means that the stock of the American Express Company is selling for $131¾ a share. This is the market price.

The quotation, "Lake Shore 4s 95⅝," means that one $100 bond of the Lake Shore Railroad, bearing 4% interest, is selling for $95⅝.

Men invest in farms, houses and lots, and other forms of property, hoping that the value of the property will increase, and that it will pay a good rate of income, as in rent. Investments are made in stocks and bonds for the same reasons.

If two investments are equally safe, the one paying the better rate of income is likely to cost the more. If a man is buying land to raise corn, he will pay more for an acre that will produce 60 bushels than for one that will produce only 30 bushels.

If two investments pay the same rate of income, the safer one may be expected to cost the more. If two bonds pay the same rate of interest, the one having the better security may be expected to bring the better price in the market.

Exercise 93

In buying stocks and bonds the brokerage, which is $\frac{1}{8}\%$ of the par value, is added to the market price to obtain the total cost. When stocks and bonds are sold, the brokerage is subtracted from the market price to obtain the amount received by the owners.

Use the quotations on page 198 in the first 13 exercises.

1. What is the cost of 20 shares of American Express?

SOLUTION. $\$131\frac{3}{4}+\$\frac{1}{8}=\$131\frac{7}{8}$, the total cost of one share including brokerage.

$20\times\$131.875=\$2637.50$, cost of 20 shares.

2. Find the cost of 40 shares of Illinois Central.

3. Find the cost of 10 shares of Pullman Car Company and 30 shares of the Massachusetts Electric Railway.

4. What is received from the sale of 50 shares of United States Steel, common?

HINT. Remember to subtract the brokerage.

5. What is received from the sale of 25 shares of the Standard Oil Company of Indiana?

6. What is received from the sale of $4000 of United States 4s, coupon?

HINT. This means that the par value of the bonds sold is $4000.

7. Can you suggest a reason why Illinois Central stock is higher than Missouri Pacific?

8. Which is quoted higher, United States Steel, common, or United States Steel, preferred? Bethlehem Steel, common, or Bethlehem Steel, preferred? Do you see any reason why common stock should be lower than preferred in one case and higher in the other?

9. How much must you pay for one $100 Rock Island bond? How much interest does it yield annually? What rate of interest is received on the money invested?

10. Find the rate of interest on the investment for each kind of bond quoted. Arrange the results in order, putting the highest first. Which kind pays the highest rate of interest on the investment? Which pays the lowest?

11. What rate of interest is charged for money in your vicinity? How many of the bonds pay as good a rate of interest on the investment?

12. Which stock is probably paying a higher rate of dividend, American Express or Pullman Car Company?

13. Can you suggest a reason for there being a difference in the prices of Lake Shore 4s and Rock Island 4s? Is there a difference in the income? How many dollars interest is received annually from a $100 bond in each case?

14. A buys 5% bonds at $124\frac{7}{8}$. What rate of interest is received on the money invested?

15. Bonds bought at $119\frac{7}{8}$ pay 5% on the investment. What rate of interest do they pay on the par value?

16. What is paid for 3% bonds that pay 4% on the investment?

17. A invests in stock which pays 5% dividends. How many shares must he buy to secure an annual income of $250? What is the par value of these shares? What will they cost at $111\frac{3}{8}$?

18. How much must be invested in Liberty Bonds, $4\frac{1}{4}\%$, bought at 96, to secure an annual income of $212.50?

19. Which is the better investment, to pay $225 for land that rents for $10 an acre, or $75 an acre for land that rents for $3 an acre? What do you understand better investment to mean here?

20. Which is better, to lend money at 5% interest, or to invest in Rock Island 4s as quoted?

21. How much is gained in buying 40 shares of stock at $89\frac{3}{4}$ and selling it at $91\frac{1}{4}$?

22. Find the gain or loss in buying 80 shares of stock at $93\frac{1}{8}$ and selling it at $93\frac{1}{4}$.

23. Stock is bought at $62\frac{3}{8}$. It is sold the same day at a gain of \$2.25 a share. What is the selling price?

24. Find the total brokerage for selling 20 shares of stock at $52\frac{3}{8}$, 25 shares at $123\frac{3}{4}$, and 60 shares at $102\frac{1}{2}$.

25. How many shares of Bethlehem Steel, preferred, can be bought for \$5000? What surplus remains?

26. Name two things upon which the prices of stocks and bonds depend.

27. State differences between stocks and bonds.

28. A newspaper report of the Winchester Arms Company during one year of the World War showed net profits of \$4,652,094 on a capital of \$1,000,000. The net profits were what per cent of the par value of the stock? During the year the stock sold for \$3000 for a \$100 share. That price is what per cent of the par value? The net profits on one share were what per cent of the par value?

29. In one of its annual yearbooks the Ætna Life Insurance Company quotes in the list of stock owned, the following :

Stock	Rate of Dividend	Par Value	Market Value
First National Bank, New York	50%	\$10,000	\$100,000
Western Union Telegraph Company	$5\frac{1}{2}$%	256,000	344,000
Chicago and Northwestern, common	7%	24,900	42,579
Chicago, Rock Island and Pacific	0	10,800	3,564

Find the market value of one share of each kind of stock, assuming the par value to be \$100. Find the amount of income from each. Find the rate of income on the market value for each. Which is the best investment, based on the rate of income?

30. During the first two years following the outbreak of the World War the following stocks had the given variations in price :

Stock	Lowest	Highest
Bethlehem Steel	26	700
Baldwin Locomotive	38	$134\frac{1}{2}$
International Harvester	79	$126\frac{7}{8}$
Western Union Telegraph Company	$53\frac{3}{8}$	$105\frac{1}{2}$

If a man had bought 10 shares of each of these stocks at the lowest price and had sold at the highest price, what would have been his gain, allowing $\frac{1}{8}\%$ brokerage for buying and $\frac{1}{8}\%$ for selling? Find for each kind of stock what per cent the gain would have been of the cost.

31. During a recent year the Chicago, Milwaukee and St. Paul Railway earned 6.75% dividends on the common stock. The par value of the common stock was $117,411,300. Find the total dividends earned.

32. Liberty Bonds, bearing $4\frac{1}{4}\%$ interest, can be bought at $93\frac{7}{8}$, brokerage $\frac{1}{8}\%$. They are due in 10 years, when they will be cashed at par. What will be the total profit on a one hundred dollar bond bought now and held for 10 years? This is equivalent to what annual rate of interest on the cost of the bond?

33. Bonds due in 3 years, bearing 6% interest, can be bought for $98\frac{5}{8}$, brokerage $\frac{1}{8}\%$. What will be the total profit on a one hundred dollar bond bought now and held till due? This is equivalent to what rate of interest on the cost of the bond?

CHAPTER XII

EXCHANGE

96. Paying bills at a distance. If you wish to pay a bill in a distant city, you may use any of these methods :

Send (*a*) money; (*b*) check; (*c*) bank draft; (*d*) post office money order; (*e*) express money order; (*f*) telegraph money order.

In choosing one of these methods one should consider its safety, its cost for both sender and receiver, and its convenience for both sender and receiver. The convenience of a method depends mainly upon the access of the sender and receiver to a bank, post office, or express office.

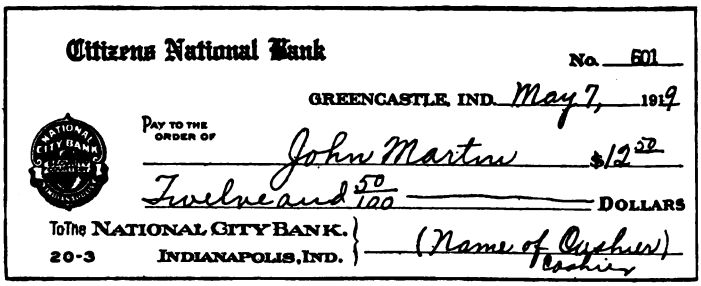
(*a*) It is seldom desirable to send the money. Coins and bills inclosed in an envelope are likely to be lost. Sending money by express or registered letter is expensive.

(*b*) If you have a bank account, it costs you nothing to send a check, and that is usually the most convenient way to pay a bill. It may cost the receiver of the check something to have the check cashed. This makes it necessary sometimes to include a small amount extra, say five or ten cents, to pay for cashing the check. The fee paid for having the check cashed is called **exchange.**

The form and uses of checks have been discussed on pages 168 and 169.

(*c*) **Bank drafts.** Banks have deposits in other banks in large cities. This money is drawn out by **bank drafts,** just as an individual draws money from a bank by a check.

If John Martin in Greencastle, Indiana, wishes to pay L. S. Ayres and Company in Indianapolis $12.50, he may go to a bank in Greencastle and get a draft such as the following:


Citizens National Bank No. 601

GREENCASTLE, IND. May 7, 1919

Pay to the order of John Martin $12.50

Twelve and 50/100 DOLLARS

To The NATIONAL CITY BANK. 20-3 INDIANAPOLIS, IND.

(Name of Cashier)

In this draft the Citizens National Bank of Greencastle is the **drawer**, the National City Bank of Indianapolis is the **drawee**, and John Martin is the **payee**. The **face** of this draft is $12.50. John Martin will indorse this draft to L. S. Ayres and Company and mail it to them. L. S. Ayres and Company will cash the draft at a bank. From this bank the draft will go to the National City Bank of Indianapolis, who will pay the draft, charge it to the account of the Citizens National Bank of Greencastle, and return it to the latter.

Banks usually do not charge for drafts issued to depositors. Other persons are usually required to pay 10 cents for drafts not greater than $100, and about .1% of the face of the draft for larger amounts.

The receiver of a draft will usually not have to pay exchange to have it cashed. Drafts on New York City banks are usually cashed without charge anywhere in the United States. If money is to be sent to a place in the central part of the United States a draft on a Chicago bank may be sent and collected without charge.

A form of draft known as a **commercial draft** is often used in collecting debts.

If John Jones of New York, N. Y., wishes to collect $89.37 from A. J. Welton of Columbus, Ohio, he may use the commercial draft found on the next page.

$89 37/100 New York July 12 1918

At sight Pay to the

Order of The Second National Bank of Columbus, Ohio

Eighty-nine and 37/100 Dollars

Value received and charge the same to account of

To A. J. Welton
No Columbus, O. } John Jones

After writing this draft, John Jones sends it to the Second National Bank of Columbus. The bank sends the draft by a messenger to A. J. Welton and requests payment. When A. J. Welton pays the draft, it is marked "Paid" and he gets it as a receipt. When the bank receives the $89.37, it deducts a commission for collecting, and sends the proceeds to John Jones. The commission is usually from .1% to 1%, with a minimum charge varying from 15 to 25 cents.

A commercial draft often accompanies a bill of goods which is sold on credit. The freight bill may be attached to a commercial draft, thus making it impossible for the buyer to receive the goods until the draft is paid. If A. C. Conley should sell G. A. Hunter a bill of goods for $256 on 60 days credit, he might send the following draft along with the bill of goods.

$256 00/100 New York Aug. 16 1918

At sixty days sight Pay to the

Order of Myself

Two Hundred Fifty-six and 00/100 Dollars

Value received and charge the same to account of

To G. A. Hunter
No Staunton, Va. } A. C. Conley

G. A. Hunter accepts this draft by writing the word "Accepted," the date, and his name across the face of the draft. By accepting the draft he becomes liable for its payment. The draft is then returned to A. C. Conley, who may hold it until it becomes due, or if he wishes his money before the draft is due, he may discount it at a bank like a promissory note.

(*d*) **Postal money orders.** A postal money order is an order by one postmaster on another to pay a stated sum of money to the person named in the order.

The fees for postal money orders payable in the United States and its dependencies are as follows :

For Orders	From $ 0.01 to $ 2.50	3 cents
	From $ 2.51 to $ 5.00	5 cents
	From $ 5.01 to $ 10.00	8 cents
	From $10.01 to $ 20.00	10 cents
	From $20.01 to $ 30.00	12 cents
	From $30.01 to $ 40.00	15 cents
	From $40.01 to $ 50.00	18 cents
	From $50.01 to $ 60.00	20 cents
	From $60.01 to $ 75.00	25 cents
	From $75.01 to $100.00	30 cents

(*e*) **Express money orders.** These orders are issued by an agent of an express company and may be cashed at any office of the company, and also at certain banks where the company has money on deposit. The rates are the same as on postal money orders.

A check payable to order, draft, postal money order, or express money order must be indorsed before it is cashed. If the payee is not known, he must be identified. The person who does the identifying usually indorses the check, draft, or money order and thus becomes liable for its payment. The process of identification is about the same for all these methods of exchange. Hence one of these ways is about as *safe from* loss by theft as another.

Exercise 94

1. Compare a check, draft, postal money order, express money order as to safety, expense, and convenience.

2. If you were paying a bill of $15 in a distant city, how would you pay it? Give reasons for your answer.

3. What is the fee for a postal money order for $4.75? For $6.20? For orders for $126.78?

4. In the draft at the top of page 205 who is the drawee? The payee? To whom is the draft first sent? Who sends it? Who pays this draft? How much does he pay? To whom does he pay it? Who gets the draft when paid?

5. John Doe of Andover, Mass., owes M. R. Steel of Portland, Maine, $39.26. John Doe buys a draft payable to his own order for $39.26, from the State Bank of Andover of which A. C. Reese is cashier, on the Fifth National Bank of Boston. The draft is indorsed in full to M. R. Steel. Write the draft and properly indorse it. John Doe pays for the draft, plus 10 cents exchange, by a check on the State Bank. Write the check.

6. What rate of exchange are you paying if you pay 10 cents for a draft for $20?

7. I get $124.85 for a check for $125. What rate is charged for cashing it?

8. L. H. Kurtz of Oshkosh, Wisconsin, buys a bill of goods for $784 from Marshall Field and Company of Chicago, on 60 days credit. Marshall Field and Company send with the bill for the goods a draft payable at 60 days sight. L. H. Kurtz accepts the draft October 18, 1918, and returns it to Marshall Field and Company. The draft is later sent to the Farmers Bank of Oshkosh for collection. On the day it is due L. H. Kurtz borrows from the bank the money to pay it and gives a note for 90 days with interest at 6% per annum from date. The bank deducts .1% for collection and remits the proceeds to Marshall Field and Company by a draft

on the National City Bank of Chicago. When the note is due, L. H. Kurtz pays it by a check on the Farmers Bank of Oshkosh. Write the two drafts, the note, and the check.

9. If you had an express money order for $13.65, what would you do to get the money?

10. If you had $327.58 deposited in a bank and wished to purchase a draft for $56, what would you do? Would you probably have to pay exchange?

11. If the second draft on page 205 is discounted September 12 at the Peoples Bank at 6%, what are the proceeds?

12. A draft for $960, dated May 17, due in 60 days, is discounted at 7%, June 11. Find the proceeds.

13. In the second draft on page 205, who are the drawer, drawee, and payee? Explain the use of this draft. When does G. A. Hunter become a party to this draft? How? If this draft is discounted at a bank, as in the last exercise, who indorses it? Who gets the proceeds? When? Who gets the discount? When? How much does the bank make out of the transaction? Who pays the draft when due? How much does he pay? Who gets the draft when paid?

14. A bank in one day collects commercial drafts for the following amounts: $89.60; $8.32; $212; $350; $3.40. The bank charges for collection $\frac{1}{4}\%$ to the nearest 5¢, with the exception that no collection fee is less than 10¢. For example, the charge for collecting a draft for $128.30 is not 32¢ but 30¢. Find the total charges for collecting the five drafts.

15. At the same rate as in the preceding exercise find the total charges for collecting drafts for the following amounts: $75; $342; $12.50; $1.10; $42.30.

16. Write a bank draft for $76.37, dated August 7, 1920, of which the Merchants Bank of Topeka, Kansas, is drawer, the Commercial National Bank of New York the drawee, and A. B. Lowe the payee.

97. Foreign money. Each country has a system of money. Foreign travel and commerce and our reading about the affairs of other countries make it useful to know something about foreign money.

Here are given the values in United States money of the principal units of value in certain foreign countries :

Country	Unit of Value	Equivalent	Approximate Equivalent
Great Britain . .	Pound (£)	$4.8665	$5
Canada	Dollar	1	1
France	Franc (fr.)	.193	.20
Germany . . .	Mark (M.)	.238	.25
Mexico	Peso	.498	.50

Table of English Money

4 farthings (far.) = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound (£)

In French money, 100 centimes (c.) = 1 franc (fr.)
In German money, 100 pfennige = 1 mark (M.)

Exercise 95

In the first 13 exercises use the approximate equivalents.

1. Find the value in United States money of £2; £1000; 1*s.*; 5*s.*; 1*d.*; 6*d.*; £50; 10*s.*; £200.

2. Find the value in United States money of 10 fr.; 5 fr.; 50 c.; 2 fr. 50 c.; 1000 fr.; 10 c.; 800 fr.

3. Find the value in United States money of 10 M.; 4 M.; 40 pf.; 8 M. 50 pf.; 1000 M.; 10 pf.; 200 M.

4. Express in French money : $1; 10¢; 25¢; 60¢; $10; $1000.

5. Express in English money : $25; 75¢; $1000; 2¢; 1¢; $2.50; $400.

6. Express in German money: \$1; \$10; 50¢; 10¢; 1¢; \$1000; \$50; 25¢; \$400.

7. A tourist paid 3 fr. 50 c. for luncheon and gave the waiter 50 c. What was the total cost in United States money?

8. What change should be received from a 10-franc piece which is given in payment for articles costing 2 fr. 20 c., 1 fr. 50 c., and 3 fr. 80 c.?

9. What change should be received from a 5-pound note when given in payment for a hat costing 10*s.*, and 6 shirts costing 7*s.* 6*d.* each?

10. A panama hat bought in Munich cost 13 M. That is how much in our money?

11. One dollar of our money is equivalent to about how much in English money? In French money? In German money?

12. The United States lent England at one time \$400,000,000. That is about how many pounds?

13. If you had \$150 changed into Mexican money, about how many pesos should you get? About how many dollars should be received for 625 pesos?

In the following problems use the exact equivalents.

14. On July 8, 1915, the Continental and Commercial National Bank of Chicago offered to pay the following amounts for foreign currency: English, \$4.73 for £1; French, \$17.75 for 100 fr.; German, \$18.75 for 100 M.; Canadian, \$.99. Find how much below the normal value, as given in the table, each of these is. Each is what per cent below normal value?

15. On October 31, 1917, there could be bought in New York for \$4.7525 a draft on London for £1. That is what per cent below normal value?

16. On the same day there could be bought for \$1 a draft on Paris for 5.73 fr. What was the cost of 1 fr.? That is what per cent below normal value?

17. An importer gets a bill of goods from France for 2560 fr. Find the cost in our money.

18. Using the exact equivalent, find the answer to exercise 12 to the nearest penny.

19. Some books imported from England cost £6 8*s*. That is how much in our money?

20. Find the cost in United States money of a bill for 7960 pesos.

21. Before the World War the postage rate on an ordinary letter was in the United States 2¢, in England 1*d*., in France 10 c., and in Germany 10 pfennige. Express each of these in cents, correct to .001¢, and arrange in order of size. Find in dollars and cents the postage on 1000 letters in each country, correct to 1¢.

22. An express check calls for \$10. What should it call for in English, French, and German money?

23. The distance from Southampton to London is 79 miles, and the railroad fares are as follows : 1st class, 13*s*.; 2d class, 8*s*. 6*d*. ; 3d class, 6*s*. 6*d*. Find the rates per mile in United States money, correct to .1¢.

24. The distance from Ostend to Brussels is 125 kilometers, and the railroad fares are as follows : 1st class, 11 fr. 80 c. ; 2d class, 8 fr. ; 3d class, 4 fr. 75 c. Find the rates per mile in United States money, correct to .1¢. The franc and centime have the same values in Belgium as in France.

CHAPTER XIII

REVIEW EXERCISES AND SPECIAL APPLICATIONS

Exercise 96. Fractions

1. Show by substituting values for the letters that $\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}$. Translate this formula into a rule. Use the rule you have just found in finding the following sums :

2. $\frac{1}{2}+\frac{1}{3}$. **4.** $\frac{1}{4}+\frac{1}{6}$. **6.** $\frac{1}{10}+\frac{1}{7}$. **8.** $\frac{1}{2}+\frac{1}{5}$.

3. $\frac{1}{5}+\frac{1}{6}$. **5.** $\frac{1}{8}+\frac{1}{2}$. **7.** $\frac{1}{3}+\frac{1}{4}$. **9.** $\frac{1}{3}+\frac{1}{8}$.

10. Show by substitution that $\frac{1}{a}-\frac{1}{b}=\frac{b-a}{ab}$. Translate this formula into a rule. Use the rule you have just found in finding the following differences :

11. $\frac{1}{2}-\frac{1}{3}$. **13.** $\frac{1}{3}-\frac{1}{7}$. **15.** $\frac{1}{6}-\frac{1}{10}$.

12. $\frac{1}{2}-\frac{1}{5}$. **14.** $\frac{1}{4}-\frac{1}{9}$. **16.** $\frac{1}{12}-\frac{1}{25}$.

17. State two ways of multiplying a fraction by an integer.

18. Multiply each of the following fractions by 3 and give the results in their lowest terms : $\frac{2}{3}$, $\frac{5}{6}$, $\frac{2}{7}$, $\frac{4}{15}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{5}{12}$, $\frac{1}{3}$.

19. State two ways for dividing a fraction by an integer.

20. Divide each of the following fractions by 4 and give the answers in their lowest terms : $\frac{8}{9}$, $\frac{12}{15}$, $\frac{4}{7}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{6}{7}$, $\frac{5}{24}$, $\frac{12}{16}$.

21. Give the following products in their lowest terms : (*a*) $\frac{2}{3}\times\frac{4}{5}$; (*b*) $\frac{1}{2}\times\frac{5}{6}$; (*c*) $\frac{8}{9}\times\frac{3}{4}$; (*d*) $\frac{7}{8}\times\frac{5}{12}$; (*e*) $\frac{4}{5}\times\frac{5}{12}$; (*f*) $\frac{2}{10}\times\frac{5}{7}$; (*g*) $\frac{a}{b}\times\frac{c}{d}$; (*h*) $\frac{x}{y}\times\frac{r}{s}$; (*i*) $\frac{5}{6}\times\frac{x}{a}$.

22. Give the following quotients in their lowest terms : (*a*) $\frac{5}{6}\div\frac{1}{6}$; (*b*) $\frac{6}{7}\div\frac{2}{7}$; (*c*) $\frac{3}{5}\div\frac{5}{7}$; (*d*) $\frac{a}{b}\div\frac{c}{d}$; (*e*) $\frac{x}{y}\div\frac{m}{n}$; (*f*) $\frac{r}{s}\div\frac{2}{3}$.

Exercise 97. Decimals

1. Reduce each of the following to an equivalent common fraction in its lowest terms : .6, .05, .25, .75, .33$\frac{1}{3}$, .66$\frac{2}{3}$, .37$\frac{1}{2}$, .87$\frac{1}{2}$, .12$\frac{1}{2}$, .6$\frac{1}{4}$, .8$\frac{1}{3}$, .83$\frac{1}{3}$.

Add :

2.	3.	4.	5.	6.	7.
.456	24.65	6.12$\frac{1}{2}$	12.33$\frac{1}{3}$	.006	.02$\frac{1}{4}$
.34	12.68	.00$\frac{1}{4}$	46.1582	.8$\frac{1}{3}$	.002$\frac{1}{4}$
.8	80.05	.56$\frac{1}{3}$	107.2$\frac{5}{6}$	.0$\frac{1}{12}$	.02$\frac{1}{4}$

8. Find the following products : (*a*) .05×.7 ; (*b*) 60×.03 ; (*c*) 100×.0$\frac{1}{3}$; (*d*) 10×4.00$\frac{5}{6}$.

9. Find the following quotients : (*a*) .001÷.01 ; (*b*) 10÷.01 ; (*c*) 30÷.05 ; (*d*) .06÷200 ; (*e*) $\frac{1}{2}$÷.02 ; (*f*) $\frac{1}{3}$÷.0$\frac{1}{3}$; (*g*) $\frac{7}{8}$÷100.

10. State a short method of multiplying by 25 ; by 50 ; by 33$\frac{1}{3}$.

11. Multiply each of the following numbers by 25 : 24, 360, 500, 42, 12, 35, 23.

12. Multiply each of the numbers in the preceding exercise by 33$\frac{1}{3}$; by 75.

13. Reduce each of the following to hundredths: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{7}{8}$, $\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{8}$, $\frac{3}{10}$, $\frac{1}{5}$, $\frac{3}{8}$, $\frac{4}{5}$, $\frac{5}{8}$.

14. Find the circumference of a circle whose diameter is 15 ft. 4$\frac{1}{4}$ in., correct to .1 in.

15. The area of a rectangle is 27.56 sq. ft., and the width is 4 ft. 2$\frac{1}{2}$ in. Find the length correct to .1 in.

16. $A=\pi r^2+\pi rl$. Find A, correct to .01, when $r=3.5$ and $l=7.2$. This is the formula for the total area of a cone the radius of whose base is r and whose slant height is l.

Exercise 98. The literal notation

1. If 3 oranges cost x cents, find the cost of each. If a oranges cost m cents, find the cost of each.

2. Find the altitude of a rectangle whose area is $4ax$ and whose altitude is a ; whose altitude is $2x$.

3. Find the total cost of 3 articles at x cents each, 20 articles at m cents each, and a articles at b cents each.

4. If a man can do a piece of work in 3 days, what part can he do in 1 day? If he can do the work in $6\frac{1}{2}$ days, what part can he do in 1 day? What part in 3 days?

5. If a man can do a piece of work in d days, what part can he do in 1 day? What part in 5 days? What part in m days?

6. A salesman begins in a new position at a salary of \$600 a year. His salary is increased \$50 a year for t years. What is his annual salary at the end of t years?

7. A passenger train is running at the rate of 12 miles an hour, and increases its speed at the rate of 4 miles an hour each minute. Find its rate after 6 minutes. Find its rate at the end of t minutes.

8. Write a formula which gives the number of dollars, N, in d silver dollars, f five-dollar bills, 20 half-dollars, q quarters, s dimes, and 27 pennies.

9. Write a formula for the number, N, of seats in a room with d double desks, n chairs, and 12 settees each seating 4 persons.

10. A piece of sheet-iron weighs 1 pound to the square foot. How many pounds will a rectangular piece $(n+2)$ feet long and w feet wide weigh?

11. Write a formula which gives the number of cents, C, in r dollars, 5 quarters, d dimes, and 12 nickels.

12. How many rectangular blocks x in. by y in. by z in. can be placed in a rectangular box a ft. by b ft. by c ft.?

Exercise 99. The fundamental operations

Combine like terms :

1. $2x-7+4x-12x+2+8x$.
2. $25-7n+5n-9+2$.
3. $3^2+4\cdot 2^3-5\cdot 3^2+2^3$.
4. $8+5\cdot 6-4\cdot 8+2\cdot 6-3\cdot 8$.
5. $.3m-7.4m+n+.03m-1.2n$.
6. $y-\frac{1}{2}x+\frac{2}{3}y+2\frac{1}{4}x-3y$.

Find the following products :

7. $a^4\cdot a^2\cdot b\cdot a\cdot b^5$.
8. $r\cdot s\cdot t\cdot r\cdot s^2\cdot t^3$.
9. $(-2)\cdot 3\cdot(-4)\cdot 1$.
10. $(-2)^2\cdot(-3)\cdot 4$.
11. $2\cdot 0\cdot(-5)\cdot(-8)$.
12. $(\frac{1}{2})^2\cdot 20$.
13. $(m-n)^2$.
14. $(a+b+c)^2$.
15. $3ab(a^2-2ab+b^2)$.
16. $.6rs(r^2+rs+s^2)$.
17. $7(200+40+8)$.
18. $(30+5)^2$.

Find the following quotients :

19. $\dfrac{6a^3b^4}{3ab^2}$.
20. $\dfrac{18r^3s^2t^6}{9rs^2t^4}$.
21. $\dfrac{4xy^2z^4}{12x^2yz^4}$.
22. $\dfrac{a^2+a}{a}$.
23. $\dfrac{a-b+c-1}{-1}$.
24. $(42m-15n+6)\div 3$.
25. $(8x^3-14x^2-2x)\div(-2x)$.
26. $(5\cdot 6^3+3\cdot 6^2-2\cdot 6)\div 6$.
27. $(4^5-12\cdot 4^3+4^2)\div 4^2$.
28. $(2^4-5\cdot 2^2+2)\div 2$.
29. $(2^3)^2=?$
31. $(a^3)^2=?$
33. $(2b)^2=?$
35. $(3x^3)^2=?$
30. $(5^2)^3=?$
32. $(ab)^2=?$
34. $(2x^2)^3=?$
36. $(-2x)^3=?$
37. $6\frac{1}{4}\times\frac{2}{5}=?$
44. $-2a\div(-\frac{1}{2})=?$
50. $\frac{2}{3}a\cdot\frac{6}{7}a-a=?$
38. $(-\frac{2}{3})^2=?$
45. $0\div\frac{3}{4}=?$
51. $(3x^2-6x^3)\div 3x=?$
39. $(-1\frac{1}{4})^3=?$
46. $0\times\frac{3}{4}=?$
52. $\sqrt{\frac{4}{9}}=?$
40. $(-.1)^3=?$
47. $\frac{1}{2}\times 0=?$
53. $\sqrt{3^2+4^2}=?$
41. $-2.3\times.04=?$
48. $0+\frac{2}{5}=?$
42. $-8\div.2=?$
49. $0-\frac{7}{8}=?$
54. $\sqrt{30^2+72^2}=?$
43. $(-\frac{3}{5})^2\div 1\frac{4}{5}=?$

Exercise 100. Equations and Formulas

Solve for x and check :

1. $3x-8+2x=x+42$.
2. $\frac{x}{3}+5=3x-\frac{1}{2}$.
3. $\frac{x}{5}+\frac{x}{6}=22$.
4. $\frac{2}{3}x-\frac{1}{2}x=3$.
5. $5x=m$.
6. $ax=r$.
7. $2x+m=n$.
8. $2x+a=a$.
9. $2x^2=50$.
10. $3x^2+9=156$.
11. $7x^2-5=65$.
12. $ax^2=b$.

13. The volume of a ring (Figure 83) is given by the formula $V=2\pi^2r^2R$. Find the volume of the ring for which $r=1$ in. and $R=4$ in.

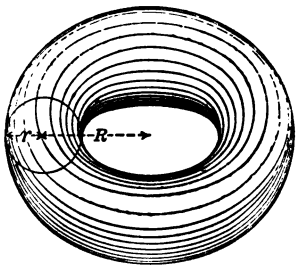


FIG. 83

14. Find the weight of the iron required to make 1000 rings in which $r=\frac{1}{4}$ in. and $R=3$ in. The specific gravity of iron is 7.86.

15. If a wrought-iron bar 2 in. deep and 1 in. wide is supported at both ends and loaded in the middle with a weight of W pounds, then the amount the bar is bent, called the deflection, is given by the formula $D=\frac{WL^3}{32\times29\times10^6}$, where W = the load in pounds, and $L=$ the length in inches. Find D if $W=2$ T., and $L=6$ ft. Find D if $W=2$ T. and $L=4$ ft.

16. The resistance of the air to the flight of a bullet is said to be given by the formula $F=.0139\,d^2(v-800)$, where F is the resistance in pounds, d is the diameter of the bullet in inches, and v is the velocity of the bullet in feet per second. In this formula v is supposed to be greater than 1100. Find the resistance of the air to a bullet which has a diameter of .3 in. and a velocity of 1500 ft. per second.

17. Find the resistance of the air to a bullet .44 in. in diameter and with a velocity of 2000 ft. a second.

Exercise 101. Ratio, proportion, and similar figures

1. Define ratio. What are the terms of a ratio called?

2. Define proportion. What are the means? What are the extremes? State a principle that is used in solving a proportion.

3. What is meant by the specific gravity of a substance?

Solve the following proportions :

4. $x : 12 = 4 : 10$.

5. $.35 : x = .07 : 16$.

6. $\frac{1}{2} : \frac{4}{5} = x : 5$.

7. $.004 : .01 = 25 : x$.

8. A liter of air weighs 1.293 g. A liter of water weighs 1 Kg. Find the specific gravity of air compared to water.

9. Find the weight of air in a room 10 meters by 8 meters by 4.5 meters.

10. Chlorine gas is 2.44 times as heavy as air. What is the specific gravity of chlorine gas compared to air?

11. A legacy of \$1200 is divided between two persons in the ratio of 2 to 3. Find how much each receives.

12. The ratio of the cement to the sand in a mixture of concrete is $1 : 2\frac{1}{2}$. How much cement and how much sand in 20 cu. ft. of concrete?

13. Define similar figures.

14. Name the corresponding parts in the similar triangles in Figures 30 and 31, p. 77.

15. In two similar triangles ABC and $A'B'C'$, $AB = 12$ ft., $BC = 20$ ft., $A'B' = 8$ ft., and $A'C' = 12$ ft. Find AC and $B'C'$.

16. Two observers at A and B see an airplane fly over the line AB. From A the angle of elevation of the airplane is 75° and from B, 68°. The distance from A to B is $\frac{1}{2}$ mile. Make a drawing to scale and find, by measuring, the height of the airplane.

Exercise 102. Practical measurements

1. What is a board foot? How many cubic inches in a board foot? How many board feet in a cubic foot?

2. If you are given the length of a stick of timber in feet, the width in inches, and the thickness in inches, how do you find the number of board feet the stick contains?

3. In a rain of $1\frac{1}{2}$ in. how many barrels of water will fall on 1 acre?

4. A lean-to roof has a span of 12 ft. and a rise of 4 ft. It is 20 ft. long. Find the number of square feet of felt roofing required to cover it.

5. Find the number of gallons of water falling on this roof in a rain of $\frac{3}{4}$ in.

6. How deep will this amount of water fill a cylindrical cistern which is 8 ft. in diameter?

7. A tin pail is 14 in. in diameter at the top and 11 in. at the bottom. How much water will fall into this pail during a rainfall of 1 in.?

8. A bushel of ear corn is commonly taken to be $2\frac{1}{4}$ cu. ft. How many bushels are there in a crib 74 ft. long and 12 ft. wide, if the corn is $6\frac{1}{2}$ ft. deep?

9. Water is flowing from a pipe which is 12 inches in diameter, at the rate of 10 miles an hour. How many gallons flow out in 10 minutes?

10. A bushel of ear corn weighs 70 lb., and a bushel of shelled corn weighs 56 lb. What per cent of the weight of a bushel of ear corn is cobs?

11. A bushel of shelled corn is 2150.4 cu. in. What per cent of the volume of a bushel of ear corn is cobs? See problem 8.

12. A 1-inch cube of metal is heated until its edge is increased to 1.004 in. By what per cent is its edge increased? By what per cent is the area of its surface increased? By what per cent is its volume increased?

13. The length of a steel rod is increased .000007 of its length for each degree of increase in temperature. By how much is the length of a steel rod 30 ft. long increased if the temperature is increased 100°?

14. Show that a round barn 60 ft. in diameter and a rectangular barn 36 ft. by 78½ ft. have practically the same floor space.

15. Find the number of board feet of siding for each of the barns in the previous problem, if it is 18 ft. to the eaves in each case. Do not consider the gables. Find the cost of the siding in each case at $35 a thousand feet.

16. The following data were taken from Bulletin No. 143 of the Illinois Agriculture Experiment Station.

The amounts of certain kinds of materials for a round barn 60 ft. in diameter and for a rectangular barn with a plank frame 36 ft. by 78½ ft. are as follows :

Material	Round Barn	Rectangular Barn
Framing timbers	13,976 ft.	19,833 ft.
Sheathing, siding, and flooring . . .	12,971 ft.	15,355 ft.
Shingles	44,000	45,000

Find the cost of the materials for each barn if the framing timbers cost $30 a thousand feet, the sheathing, siding, and flooring $30 a thousand feet, and the shingles $5 a thousand. The cost of materials for the rectangular barn is what per cent more than the cost of materials for the round barn? Find the cost of a concrete floor for each at 18¢ a sq. ft.

Exercise 103. Percentage

1. Read the per cents equivalent to the following : .25, .07, 6, 35, 489, .0003, .00½, .07⅓, .8⅓, 6.502, .5⅔.

2. Read the equivalent per cents : ½, ⅔, ⅘, ⅚, ⅜, ¾, ⅝, ⅞, ⅛, ⅓, 1/10, ⅖, 1/12, 1/16.

3. Read the equivalent decimal fractions : 17%, .6%, 9.4%, 200%, 1568%, .001%, ⅓%, ½%, ⅞%, 3.004%.

4. A grocer buys eggs at 32¢ a dozen and sells them for 36¢ a dozen. What per cent of the cost does he gain?

5. If he buys eggs at 20¢ a dozen and sells them at the rate of 10 for 20¢, what per cent of the cost does he gain?

6. A buys a house for $2450. He sells it to B, losing 10% of the cost. B gains 10% of what the house cost him by selling it to C. Did C pay the same for the house as A? If not, how much more or less?

7. A hat is marked to sell at 25% above cost. The marked price is $6. The hat is sold for $5.50. Find the per cent of gain or loss on the cost.

8. An article that costs $9 is sold for $11.50. Find the per cent of gain on the cost. Find the per cent of gain on the selling price.

9. At a fire sale the prices were marked down 12½%, to the nearest 5¢. Find the sale prices of articles which before the fire had sold for the following prices : $8 ; $12.50 ; $6 ; $25 ; $3.25 ; $7.75 ; $2.50.

10. An accident policy for $5000 costs $29 a year. What is the rate of premium?

11. In July, 1865, it took $100 in paper currency to buy $39 in gold. The paper currency was worth what per cent of its face value in gold? How much currency was needed to buy $100 worth of gold?

12. From 1860 to 1865 prices increased 117%, that is, an article that cost $1 in 1860 cost $2.17 in 1865. At that rate, an article that cost $1 in 1865 cost how much in 1860?

Exercise 104. Insurance

The data in the following problems were taken from a report of the Superintendent of Insurance of Illinois for a recent year.

1. The following table gives the amount of insurance, the premiums paid, and the losses paid on certain classes of buildings. These figures are for the whole state.

Kind of Building	Amount Insured	Premiums	Losses
Barns and stables (not on farms)	$11,474,695	$121,819.21	$118,490.28
Brick dwelling, combustible roof	62,454,476	579,545.07	134,931.95
Brick dwelling, non-combustible roof	148,724,335	1,202,365.52	301,873.46
Frame dwelling, combustible roof	261,960,252	2,422,635.00	1,160,123.18
Clothing stores	2,879,078	44,965.14	15,967.80
Public garages	4,516,681	112,269.69	24,483.65

Find the rate of insurance per $100 on each class of buildings. Do you see any causes for the differences?

Find the ratio of the losses to the premiums for each class of buildings. On which class of buildings is the insurance the most profitable for the insurance companies? What per cent of the premiums was returned to the policy holders in each case?

2. The average rate of insurance in Norway in a certain year was 11.7¢ per $100. The average rate in the United States in the same year was $1.037 per $100. Find the cost of a policy of $5000 in each country at these rates. Do you know some of the probable causes for this difference in rates?

3. Find the rates of insurance in your neighborhood on the classes of buildings mentioned in problem 2. How do these rates compare with the rates found in that problem?

Exercise 105. Stocks and bonds

1. In its annual report for a recent year the United States Steel Company showed gross sales of $1,205,000,000, which were 41% more than for the preceding year. Find the sales for the preceding year to the nearest million dollars.

2. The net earnings of the United States Steel Company for this year, after deducting Federal taxes, were $224,219,564. The Federal taxes were $223,465,435. What was the total of the net earnings including Federal taxes? What per cent of the total net earnings was paid for taxes?

3. At this time United States Steel common was selling at 90 and preferred for 109. Can you suggest any probable cause for this difference in price?

4. The following bond quotations are from the New York stock market:

U. S. Liberty Loan 3½s	99
U. S. Liberty Loan 4s	96.76
U. S. Liberty Loan 4¼s	95.58
Anglo-French 5s	90¼
City of Paris 6s	85

Find the rate of income on the investment paid by each kind of bonds bought at the price quoted, brokerage ⅛%. Which investment yields the highest rate of income?

5. Find the cost of 10 $100 bonds of each of the kinds mentioned in the previous exercise, allowing ⅛% brokerage.

6. Find the cost of $1000 worth of U. S. Liberty 4¼s at the price quoted above, brokerage ⅛%.

7. How much must be invested in City of Paris 6s as quoted above to secure an annual income of $750, brokerage ⅛%?

8. Name some differences between stocks and bonds.

9. Find the gain if 1400 shares of Union Pacific are bought at 118¾ and sold for 120½, brokerage ⅛% for buying and the same for selling.

98. Investments. The principal things to be considered in making an investment are its security and the rate of income it yields.

Many investments are advertised to yield large rates of income. These investments are often unsafe. If they offered good security and high rates of income, it would probably not be necessary to advertise them. But many persons with small sums to invest are attracted by the large incomes promised by "get-rich-quick" schemes, and invest in the hope of getting something for nothing.

Exercise 106

The investments offered in the first 8 of the following exercises were found in the advertisements or the market reports of daily newspapers. At the time these investments were offered money could be put out at interest with good security at 6%. State which of these are probably safe investments, and which offer as good or a better rate of income than lending money at 6%.

1. An oil company stock, par value \$1 a share, is offered at \$1.50 a share, and promises dividends "at the rate of 24% annually on par."

2. Another oil company offers "oil stock that pays 12% a year."

3. "I am in a position *right now* to deliver shares of —— Motor Car Corporation at a price which affords the buyer an immediate profit of 15 per cent."

4. Farm mortgages bearing 6% interest are offered for sale.

5. "50% profit. Established manufacturing company offers *a profit of* 50% and return of principal within 12 months to a limited number of investors."

6. United States $3\frac{1}{2}$% bonds may be bought at par.

7. Four per cent interest will be paid on money deposited in savings banks.

8. "Old, successful company, with a 30-year record of steady, consistent growth. Dividends averaging 36% per annum actually paid for the past 10 years."

9. A man lends money at 7%. The taxes are 2.2%. What is his net income on $2500?

10. In a certain year the Standard Oil Company of Kentucky made profits amounting to $32.78 a share. The shares were selling for $300 each. What rate of profits was being made on the price of the stock?

11. Suppose that you had $1000 to invest. Find three ways in which it might be safely invested in the community in which you live. If you do not know three ways, ask a business man.

12. How much larger net income will one receive from $2550 invested in 4¼% Liberty Bonds at par which are non-taxable, than from the same amount loaned at 6% in a city whose tax rate is 2¼%?

13. Find advertisements of investments in the daily newspapers and bring them to class with you. Discuss the rate of income and the security of each. What do you consider a good rate of income? If you can, find one that offers a good rate of income with good security; one that offers good security but with a low rate of income; and one that offers a high rate of income but of whose security you are in doubt.

99. Building and loan associations. A building and loan association that is well managed and properly inspected offers a good opportunity for the investment of savings.

Building and loan associations are stock companies.

Issues of new stock are made according to the demands of the business. In some associations new issues are made several times a year.

This is an example of a stock certificate of a building and loan association.


EACH SHARE ENTITLES THE HOLDER TO A LOAN OF $100.

No. ..... Series ..... Class A

Shares

Charleston

$

Homestead and Loan Association

ISSUED .....

This is to Certify That .....

is entitled to ..... Shares of the Capital Stock of the

Charleston Homestead and Loan Association

of Charleston, Coles County, Illinois, transferable in person or by attorney only, in the presence of the Secretary, and only on the books of said Association. Payments begun .....

Seventy-five cents payable monthly on each share.

..... Secretary

..... President.

If you should take ten shares of the stock of this association, you would receive such a certificate. You would agree to pay 75¢ a month dues on each share. How much would you pay each month? How much would you pay each year?

The dues on building and loan stock are sometimes paid weekly. There are other methods of payment also.

Building and loan stock offers a good method for the investment of savings that accumulate weekly or monthly, as they can be paid in and begin drawing interest at once.

The dues that are paid to the association are lent to persons who wish to borrow money. The interest on these loans goes to make up the profits of the stockholders.

The par value of each share called for in the above certificate is $100. This stock is said to **mature** when the sum of the dues and the profits amounts to $100 on each share. When the stock matures, the stockholder will return the stock certificate to the office of the association and will then receive $100 for each share he holds.

Exercise 107

1. Do you know of a building and loan association? Are investments in its stock considered safe? What rate of interest does it pay its stockholders? How often are dues paid? How much are the dues on a share? Do any of the leading business men in your community own stock in this association?

2. The dues on Class A stock in a building and loan association are 75¢ a month on each share. This stock usually matures in 100 months, and is worth $100 a share when it matures. How much has been paid in dues on a share in that time? How much interest has accumulated?

3. Class B stock in this company, worth $100 a share at maturity, matures usually in 80 months. The dues are $1 a share each month. How much is paid in dues on a share before maturity? How much interest accumulates on a share?

4. Raymond Willis, who is 14 years old and is entering high school, saves about $2 a week and decides to subscribe for 5 shares of building and loan stock, paying 20¢ a week dues on each share. One year later he subscribes for 2 more shares, and at the end of another year he subscribes for 3 more shares. When he goes to college, which is four years after he subscribed for the first stock, he cashes all of his building and loan stock. On the first stock he receives $46.60 a share, on the second, $33.81 a share, and on the third, $21.92 a share. How much did he receive for all of his stock? How much dues had he paid? How much profit had he made?

5. John Capper pays $7.50 a month for 100 months on 10 shares of building and loan stock. He then draws out $1000. How much has he paid in? The average time that his payments have drawn interest is 50 mos. What rate of interest has his money earned?

100. Parcel post. The parcel post rate depends upon the weight of the parcel and the distance that it is sent. The distance is indicated by zones. The first zone includes places not more than 50 miles distant ; the second zone, places from 50 to 150 miles distant ; the third, 150 to 300 ; the fourth, 300 to 600 ; the fifth, 600 to 1000 ; the sixth, 1000 to 1400 ; the seventh, 1400 to 1800 ; the eighth, over 1800 miles.

Parcel post matter may not exceed 84 in. in length and girth combined. The limit of weight is 70 lb. for parcels mailed to be delivered within the first or second zones, and 50 lb. for all other zones.

The rate in cents for sending matter by parcel post is :

In the first or second zone, 4 more than the number of pounds.
In the third, 4 more than twice the number of pounds.
In the fourth, 3 more than four times the number of pounds.
In the fifth, 2 more than six times the number of pounds.
In the sixth, 1 more than eight times the number of pounds.
In the seventh, 1 more than ten times the number of pounds.
In the eighth, twelve times the number of pounds.

Exercise 108

1. Make a formula to show the cost of sending a parcel to each of the zones. Let r be the cost in cents and w the weight of the parcel in pounds.

2. Find the cost of each of the following : (*a*) Sending 6 lb. to the 3d zone. (*b*) Sending 12 lb. to the 6th zone. (*c*) Sending 20 lb. to the 5th zone.

3. Find the cost of sending each of the following : (*a*) 17 lb., 182 mi. (*b*) $12\frac{1}{2}$ lb., 80 mi. (*c*) 25 lb. from New York to Chicago, 909 mi. (*d*) 15 lb. from Denver to San Francisco, 1668 mi.

4. Make a table which gives the cost of sending each of the following weights to each of the zones : 1 lb., 2 lb., 3 lb., 4 lb., 5 lb., 10 lb., 15 lb., 20 lb., 25 lb., and 30 lb.

101. Light and ventilation in schoolrooms. Crowded and badly ventilated schoolrooms are the cause of much listlessness, restlessness, and inattention. In every schoolroom there should be liberal space for each pupil and plenty of fresh air. Insufficient light causes defective eyesight.

Exercise 109

1. Authorities on school hygiene state that a schoolroom for 40 pupils should contain at least as many cubic feet as a room 32 ft. by 24 ft. by $12\frac{1}{2}$ ft. This is how many cubic feet per pupil?

2. According to this standard how many pupils could be accommodated in a room 30 ft. by 22 ft. by 12 ft.?

3. Measure your schoolroom. Does it give sufficient air space for each pupil according to the above standard?

4. It is estimated that there should be a supply of 2500 cu. ft. of fresh air each hour for each pupil in an eighth grade room. The fresh air comes into an eighth grade room through a ventilator shaft 16 in. by 15 in. at the rate of 12 ft. per second. Is this supply large enough for 30 pupils?

5. What should be the area of a ventilator to supply 40 children with fresh air if the air moves at the rate of 10 ft. per second?

6. For a schoolroom to be well lighted it is estimated that the area of the windows should be from $\frac{1}{6}$ to $\frac{1}{4}$ of the area of the floor. In certain cities the allowance is $\frac{1}{5}$. The floor of a certain schoolroom is 32 ft. by 24 ft. It is lighted by 3 windows each 3 ft. $3\frac{1}{4}$ in. by 9 ft. $2\frac{1}{2}$ in. The area of the windows is what per cent of the area of the floor? Is the room well lighted according to the standard given?

7. A schoolroom $20' \times 30'$ is lighted by 8 windows each 2 ft. 6 in. by 5 ft. 2 in. The area of the windows is what per cent of the area of the floor? Is the room well lighted according to the standard given above?

102. A boy's dairy project. Would you like to pay your own way through the junior high school? Suppose you try the project by which Tom Winters paid his own way and finished with a nice little balance, too. I shall give you the facts about Tom's project, and you may do the figuring to find out how much he made.

Exercise 110

Tom rented a 10-acre pasture at the edge of town, agreeing to pay \$10 a month for it and not to put in it more than 4 cows. He then got his agriculture teacher to help him select 4 good dairy cows, 2 Holsteins and 2 Jerseys. One Holstein, No. 1, was just fresh and giving 42 lb. of milk daily ; the other, No. 2, had been giving milk for 6 months and was then giving 28 lb. daily. No. 3, a Jersey, was fresh and giving 32 lb. daily, while No. 4, the second Jersey, had also been giving milk for 6 months and was then giving only about 14 lb.

1. Before buying them he had each cow's milk tested, with the following results: No. 1, 2.5% butterfat ; No. 2, 3.6% ; No. 3, 5% ; No. 4, 4.3%. Estimating that the fresh cows would decrease about 40% in milk yield after 6 months, what was the daily yield of butterfat of each cow 6 months after becoming fresh?

2. If the value of a cow were based solely on her yield of butter fat 6 months after becoming fresh and cow No. 1 is valued at \$120, what is each of the other three worth?

3. Tom was planning to sell milk by the quart, so the quantity of milk was quite as important as the quality, or amount of butter fat. If he had considered quantity of milk only and had valued cow No. 1 at \$150, at what price should he have valued each of the others?

4. Tom borrowed the money from the bank with his father as security and paid \$135 for cow No. 1, \$160 for No. 2,

$150 for No. 3, and $120 for No. 4. The note he gave at the bank was dated Sept. 15, 1916, to run for 6 months, at which time the interest at 7% must be paid and a new note drawn. Write the first note he gave. How much interest did he have to pay when his note became due?

5. Tom found 2 customers who agreed to take 3 qt. each daily, and 4 to take 2 qt. each daily, if he would agree to mix it so that it would contain not less than 4% butter fat. Four other customers were to take 2 qt. each and 6 customers to take 1 qt. each, with the agreement that it should contain not less than 3% butter fat. The 4% milk he sold at $12\frac{1}{2}$¢ a quart, the 3% milk at 10¢. He then found other customers to take 2 qt. of milk daily at 10¢ a quart with no special agreement as to quality. His father agreed to take whatever surplus he had daily at 30¢ a gallon. To get 4% milk how much must he take from cow No. 1 to mix with all the milk from No. 3? If he mixes all the remainder of the milk, will it test higher or lower than 3%? How much? Use amounts of milk given at the time of buying the cows.

6. One quart of milk weighs 2.18 lb. The milk record of each cow by months was as follows beginning with September:

	9	10	11	12	1	2	3	4	5	6	7	8
No. 1	608	1143	1066	987	892	817	752	729	812	732	512	268
No. 2	806	764	648	567	392	117	1083	1023	1045	987	918	842
No. 3	70	896	841	786	744	708	624	563	618	569	438	206
No. 4	515	420	390	330	184	105	826	794	804	773	694	584

On Feb. 1 he had to give up one customer taking 2 qt. of 10¢ milk daily. Why? How much milk did Tom sell to his father that year? How much did he get for all his milk?

7. What was the annual yield of milk from each cow? Of butter fat? He could have sold the butter fat at 45¢ a pound. Would it have paid him better than to sell the milk?

8. What price must he receive for butter fat to equal 12½¢ a quart for 4% milk? To equal 10¢ a quart for 3% milk?

9. If 3% milk sells for 10¢ a quart, for how much ought 4% milk to sell, if the price were based on the amount of butter fat only?

10. Tom's expenses were almost entirely for feed, since he did all the work of milking, caring for the milk, and delivering it. You have already been told his expense for pasture. The following table gives the daily ration for each cow :

	While on Pasture from May 1 to Oct. 31		During the Winter from Nov. 1 to April 30	
Holstein	Oats	5 lb.	Oats	4 lb.
	Bran	3 lb.	Bran	3 lb.
	Corn	6 lb.	Corn	4 lb.
	Cottonseed meal	1 lb.	Cottonseed meal	1 lb.
			Alfalfa	10 lb.
			Corn stover	8 lb.
Jersey	Oats	4 lb.	Oats	4 lb.
	Bran	2 lb.	Bran	2 lb.
	Corn	6 lb.	Corn	4 lb.
	Cottonseed meal	½ lb.	Cottonseed meal	½ lb.
			Corn stover	6 lb.
			Alfalfa	10 lb.

He bought the oats at 58¢ a bushel, the corn at 96¢ a bushel, the bran at $1.65 a hundred, the cottonseed meal at $3.80 a hundred, the alfalfa at $21 a ton, and the corn stover at $8 a ton. What was his total cost for the feed of the cows for the year?

11. His other expenses for bottling the milk, for milking utensils, veterinary bills, etc., amounted to $127.50. How much did he have left to pay him for his labor and the interest on the money he had borrowed to buy the cows?

103. A boy's project with capons. In his class in agriculture Henry Jasper had been taught how to feed young capons for the market. His back yard contained a well-lighted shed $8' \times 10'$ in a lot $20' \times 30'$. He decided to buy and feed 30 capons. His father was willing to lend him the necessary capital for 6% interest.

Exercise 111

1. Henry estimated that the birds could be bought, weighing about 2 pounds each, at 15¢ a pound. He needed some poultry netting, which would cost 4¢ a foot for 50 feet and he estimated that the feed the capons would eat would cost about $25. He decided to borrow enough to have a margin of $6 for unforeseen expenses. Write the note which he gave his father, Henry Jasper, on Sept. 2, 1916, for 9 mo.

2. His account with the capons showed the following items:

EXPENDITURES

Sept.	7.	To Bob White for 14 capons, 26½ lb. at 14¢
"	3.	100 lb. bran at $1.75, 50 lb. scratch feed at $4.50 a hundred, and 100 lb. oats at $2.05
"	4.	200 lb. wheat at $4 a hundred
"	8.	To Alfred Busbee for 16 capons, 34 lb. at 14½¢
"	15.	Poultry wire, 52 ft. at 4¢ a running foot
Nov.	16.	100 lb. cracked corn at $3.75
Dec.	20.	3 bu. shelled corn at $1.20 a bu.
		2 bu. oats at $.80 a bu.
Jan.	29.	4 bu. shelled corn at $1.20 a bu.
Feb.	1.	2 gal. buttermilk at 18¢
"	10.	3 gal. buttermilk at 18¢

RECEIPTS

Dec.	20.	H. A. Barker, 1 capon, 7½ lb. at 28¢
Jan.	3.	Roscoe McClintock, 7¾ & 8 lb. at 30¢
"	5.	M. H. O'Neil, 8¾ lb. at 30¢
"	29.	R. S. Butcher, 9¾ lb. at 32¢
Feb.	2.	H. A. Barker, 7½ & 9 lb. at 33¢
"	3.	Hal Parkhurst, 6½ & 8¼ lb. at 33¢

Feb. 3. R. McClintock, 7¾ & 8½ lb. at 33¢ .....
" 10. Mrs. A. Smith, 10½ lb. at 35¢ .....
" 10. R. S. Butcher, 7¾ & 8 lb. at 35¢ .....
" 12. R. Hamer, 10½, 9 & 7½ lb. at 35¢ .....
" 17. Roy Mills, 9¼ & 7¼ lb. at 35¢ .....
" 20. Poultry dealer, 86¼ lb. at 32¢ .....

How much did receipts exceed expenditures?

3. He repaid his father's loan on Feb. 23, 1917. How much did he pay his father?

4. What was Henry's per cent of profit on the capital invested? Include interest as expense. Estimating that he spent about ½ of an hour a day in caring for and selling his capons, how much did he earn an hour for his work?

104. The United States Survey. In the larger part of the United States the land is laid off and described according to the plan of the United States Survey.

In making this survey a **principal meridian** and a **base line** at right angles to it are first located. The land is then laid off in six-mile squares by lines parallel to the principal meridian and the base line. These squares are called **townships**. A row of townships running north and south is called a **range**. The ranges are numbered east and west from the principal meridian. The townships are numbered north and south from the base line. See Figure 84. The township marked *A* is described as township 3

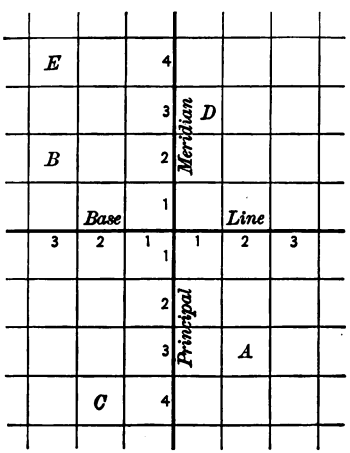


FIG. 84

south, range 2 east; the township marked *B* is described as township 2 north, range 3 west.

Each township is divided into 36 squares, each one mile on a side. Each square is called a **section.** The sections are numbered as in Figure 85.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

FIG. 85

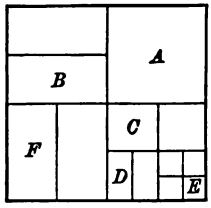


FIG. 86

Each section is divided into halves and quarters, and each of these parts may be divided into halves and quarters.

In Figure 86, the part of the section marked *A* is the northeast quarter; *B* is the south half of the northwest quarter; *C* is the northwest quarter of the southeast quarter. These descriptions may be abbreviated. Thus the description of *C* as abbreviated is the N. W. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$.

Exercise 112

1. Give the description of the township lettered *C* in Figure 84, also of *D*; of *E*.

2. In Figure 86 give the description of *F*; of *D*; of *E*.

3. Make a drawing and locate in a section the S. E. $\frac{1}{4}$; the N. $\frac{1}{2}$ of the S. E. $\frac{1}{4}$; the N. W. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$.

4. Make a drawing and locate in a section the E. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$; the S. E. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$.

5. How many acres in a section? In a quarter section?

6. Find how many acres there are in each of the tracts mentioned in exercise 3.

7. Find how many acres there are in each of the tracts in exercise 4.

8. A farm consists of the N.E. $\frac{1}{4}$, the N. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$, and the S. W. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$ of section 14. How many acres in the farm? Make a drawing and locate this farm in a section. How much is this farm worth at \$110 an acre?

9. The owner of the farm mentioned in the previous exercise buys the W. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$ of section 13. Does this adjoin the land he already owns? Make a drawing to show your answer. How much is this land worth at \$85 an acre?

105. A farm project. The members of an eighth grade class consisting of ten pupils decide to go into business together. They plan to buy a farm in which each is to have an interest and a voice in its management. They ask a banker and a leading member of a grocery firm whether they should organize a stock company or a partnership, and both agree that for their purposes a stock company is better. What are the reasons given? (See page 193.)

TO THE TEACHER. Instruction should be given at this point on the steps necessary to the formation of a corporation.

Exercise 113

Buying the farm. The class decides to form a corporation. Before it is possible to determine the amount of capital stock to be issued the class must estimate the amount of capital needed to buy and equip the farm. One member of the class is chosen as business agent and is instructed to locate a desirable farm. The class finally decides to secure an option on a farm located as follows: The west half of the southeast quarter of Section 16 in Township 3 north and in Range 7 west of the Second Principal Meridian. In the

option this description reads as follows : The W. $\frac{1}{2}$ of S.E. $\frac{1}{4}$ of Section Sixteen (16) Township Three (3) North, Range Seven (7) West.

1. How many acres in the farm?

2. Draw a plat of the township in which it lies, numbering the sections.

3. Draw on a larger scale a plat of the section in which this farm lies and show its location. In this county every section has a road around it. Show where the road touches this farm. What is the length of the road touching the farm?

4. There is a school located in the northeast corner of Section 17. How far will the school be from the house on this farm if the house is located in the southeast corner of the farm?

5. What will be the shortest distance through the fields from the house to the school?

The class decides to buy the farm at \$95 an acre. They learn that the tax rate on land in this township is 1.1% and that the farm is assessed for taxation at two-thirds the price they pay for it. They also learn that the road along the front of the farm is to be improved at a cost of \$8000 a mile. The larger part of this improvement is to be paid for from the auto-license fund, the landowners paying as special road tax only 6¢ an acre annually until the road is paid for.

6. What will be the total cost of improving the road in front of this farm?

7. What will be the road tax to be paid by this farm the first year? The total tax?

Estimating the capital needed. In order to determine the amount of capital stock to be issued by their company the class decides to make an estimate of the capital needed

to buy and conduct their farm for the first year. Their estimate is as follows :

Cost of the option	$ 760
Purchase price of farm less option	
Road improvement tax	
Fencing	500
Tenant house	2000
Drainage	220
Barn and other buildings	2200
Live stock	1000
Labor	500
Equipment	1400
General tax	
Total	

The class therefore decides to issue capital stock for 10% more than the estimated expenditures to the nearest one hundred dollars.

8. Compute the omitted estimates of expenditures, also the total estimate.

9. Compute the number of shares of stock of par value $100 to be issued.

The company is formed, B is chosen president of the company and manager of the farm, and the farm is bought.

Improving and equipping the farm. The manager at once starts the building of the boundary fences.

10. He hires 2 men for this work, paying them $2 a day each. It takes them 8 days to finish setting the posts. The posts are set a rod apart. He might have had the posts set at 5¢ a post. How much did he gain or lose by hiring the men by the day?

11. The posts cost 26¢ each and the woven wire fencing costs 48¢ a rod. The men spend 6 days in putting up the wire. What is the total cost of the boundary fence?

12. The owners of adjoining farms pay half the cost of

the division fences. What is the total cost to this farm of its boundary fences?

13. The manager finds that there must be a string of tile from the northeast corner of the farm to a point 20 rods from the road on the west line fence. This is a branch tile, and he uses 5-inch tile for the purpose. These tile are 12 inches long and cost $20 per M. What is the total cost of the tile?

14. He hires the tile hauled for $2 a load. Four hundred tiles are hauled at a load. He pays $35 a thousand for laying the tile. What is the total cost of the drainage?

15. The manager next secures the floor-plans for a five-room cottage. Complete plans and specifications are submitted to each of three contractors whose bids were $2375, $2000, and $1950. The bid of $2000 is accepted because of greater confidence in the contractor. The cost of the house is what per cent of the cost of the land?

16. How can the contractor get square corners for the foundation? What method can he use for making the front-line foundation parallel to the road? How can he be sure to make the walls of the foundation plumb?

17. While the house is being built the manager hires a man to drill a well. He pays 80¢ a foot for drilling the well under a contract which guarantees to furnish a satisfactory well of water. Good water is found at a depth of 85 ft. What is the cost of drilling the well? The casing for the well costs $25, a gasoline engine to pump the water costs $90, two water tanks for live stock cost $30, and a pressure tank installed to furnish water for the house costs $85. What is the total cost of this water system?

18. The manager also decides to build a cistern to have a capacity of 125 barrels. It is to be a cylindrical cistern. Select a convenient depth and compute the diameter so that the cistern may have the desired capacity. He hires a man

to dig this cistern at 30¢ a cubic yard. It costs 15¢ a square yard to put a wall in the cistern. The pump costs $15 installed. What is the total cost of the cistern?

19. The men who build the house also build the barn at a cost of $1200, the implement shed for $250, the chicken house for $96, the garage shop for $150, the hog house for $215, and the corncrib for $100. Find the total cost of all the buildings.

20. For house yard, garden, barnyard, and orchard he fences off a four-acre tract in the southeast corner of the farm having a south front of 20 rods. What is the cost of fencing the additional two sides of this lot at the same rate as the boundary fence cost him?

21. At the south end of the farm a pasture containing 12 acres is inclosed by a fence extending east and west. How far back from the front road is this fence put?

22. The remainder of the farm is cut into three fields. First a twenty-four acre field is cut off the east side by a fence running north and south extending from the pasture fence to the north boundary fence. The remaining forty-acre field is cut into two equal parts by a fence running east and west. These fields are numbered as follows : The north twenty acres, A ; the south twenty acres, B ; the twenty-four acre field, C ; the pasture, D. Draw to scale a plat of the farm and compute the cost of this additional fencing at the same rate as the boundary fence.

23. Find the total cost of the farm and the improvements mentioned.

Live stock and implements. The manager now buys two work horses, one for $110 and the other for $130, also a brood mare for $175, five milk cows for $110 each, and three brood sows at an average price of $38 each.

He also buys the following farm implements :

1 sulky plow	$45
1 two-horse wagon	90
1 disk	50
1 corn planter	60
1 spike-tooth harrow	20
1 grain drill	60
2 sets of harness	125
Garden implements, etc.	40
Tools, pitchforks, etc.	50

24. How much does he spend for live stock and implements?

25. What is the investment per acre in farm implements? In live stock? In fencing?

Expenses and returns. The first year field A is planted in spring wheat, field B in oats, and field C in corn.

26. The manager pays the tenant $40 a month, furnishes him a garden, permits him to keep a cow in the pasture and to keep 50 chickens for the tenant's own profit. The manager also finds it necessary to hire additional labor, which amounts to one man working for thirty days at $2.25 a day. What was the total cash outlay for this farm labor?

27. To seed his fields he buys $3\frac{1}{2}$ bu. of seed corn at $4 a bushel ; 40 bu. of oats at 70¢ a bushel; 24 bu. of wheat at $1.50 a bushel ; $2\frac{1}{2}$ bu. of clover seed at $12 a bushel to be sowed on field A. Find the total cost of seeding these fields. How much seed of each kind does he use per acre?

28. The manager also finds that the soil is sour and decides to put on limestone in order to increase the crop of clover. Before seeding the spring wheat he puts on three tons of limestone per acre at a cost of $1.60 per ton, delivered at his station. Hauling the limestone costs $18. What is the total cost of the addition of limestone to the wheat field?

29. Besides the pasture which they ate, the feed for the *five cows for* the first year costs the manager $68 each. Eight

acres of the 12 acres of pasture should be charged against the five cows at $8 an acre. What is the total cost of keeping the five cows the first year?

30. The milk production the first year is as follows :

Cow No. 1, 6200 lb. testing 4% butter fat.
Cow No. 2, 4800 lb. testing 4.2% butter fat.
Cow No. 3, 3900 lb. testing 4.4% butter fat.
Cow No. 4, 7000 lb. testing 3.8% butter fat.
Cow No. 5, 5200 lb. testing 5% butter fat.

This milk is skimmed and the butter fat sold to the local creamery. The price for butter fat during the first year averages 48¢ a pound. What is the total income for butter fat? Assuming that $\frac{1}{6}$ of the milk is hauled away from the farm in the form of cream, what is the total amount of skimmed milk left on the farm for feeding purposes? What is it worth at 40¢ a hundred pounds?

31. Of the calves from the five cows, one dies, one is sold for veal for $16, and three heifer calves are kept. Assuming that these calves are worth $25 each, what is the total profit from the five cows exclusive of labor charges?

32. The field of corn yields 44 bu. per acre, the oats 48 bu. per acre, and the wheat 22 bu. per acre. The corn is sold for 85¢ a bushel, the oats for 50¢ a bushel, and the wheat for $1.10 a bushel. What is the total income from these fields?

33. The three brood sows raise 24 pigs. They are allowed to run on pasture and are fed the skim milk from the farm during the summer. In the fall they are fed a ration of skim milk, tankage, and corn. Besides the skim milk they use 500 lb. of tankage and 280 bu. of corn. The tankage is worth $96 a ton and the corn 85¢ a bushel. The pigs are sold Dec. 15, their average weight being 250 lb. They are sold at $8.75 a hundred pounds. Assuming that the cost of the feed is 85% of the total cost of these hogs, what is the total profit on them?

The dividend. When the stock company was formed, 170 shares of capital stock were issued. A took 35 shares, B 22 shares, C 15, D 12, E 10, F 14, G 13, H 24, I 16, J 9. At the close of the year the manager takes an inventory of the farm, that is, he makes a list of the farm stock, equipment, buildings, etc., and places a present value upon them, allowing for deterioration during the year. To the value of the property shown by this inventory he adds the cash which represents the difference between receipts and expenditures for the farm. From this total he deducts the original capital invested. This difference is the profit from the year's business, and may either be kept by the company for further investment or be distributed to the stockholders as a dividend.

34. This year the directors of the company decide to distribute $1250 of the surplus as dividend. How much will each stockholder receive?

TABLES

Length

12 inches = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods or 5280 ft. = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
$31\frac{1}{2}$ gallons = 1 barrel (bbl.)
2 barrels = 1 hogshead (hhd.)

Dry Measure

2 pints = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)
365 days = 1 common year
366 days = 1 leap year

Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)

Values

10 mills = 1 cent (¢ or ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)

English Money

£1 (one pound) = 20 shillings = $4.8665

French Money

1 franc = 100 centimes = 19.3¢

German Money

1 mark = 100 pfennigs = 23.8

Miscellaneous

1 gallon = 231 cu. in.
1 cu. ft. = $7\frac{1}{2}$ gal., nearly
1 bu. = 2150.42 cu. in. = $1\frac{1}{4}$ cu. ft., nearly
1 bushel of ear corn = $2\frac{1}{4}$ cu. ft., nearly
1 cu. ft. of water weighs $62\frac{1}{2}$ lb.
1 qt. of milk weighs 2.18 lb.
$\pi = 3.1416$, or $3\frac{1}{7}$, nearly
1 bu. shelled corn weighs 56 lb.
1 bu. ear corn weighs 70 lb.
1 bu. wheat weighs 60 lb.
1 bu. potatoes weighs 60 lb.
1 bu. rye weighs 56 lb.
1 bu. oats weighs 32 lb.
1 bu. apples weighs 44 to 50 lb.
1 barrel flour weighs 196 lb.

METRIC TABLES

Length

100 centimeters (cm.) = 1 meter (m.)
1000 meters = 1 kilometer (Km.)

Area

10,000 square centimeters (cm.^{2}) = 1 square meter (m.^{2})
1 m.^{2} = 1 centare (ca.)
100 m.^{2} = 1 square dekameter = 1 are (a.)
100 ares = 1 hectare (Ha.)

Volume

1,000,000 cubic centimeters (cm.^{3}) = 1 cubic meter (m.^{3})

Capacity

100 liters (l.) = 1 hektoliter (Hl.)

Weight

1000 grams (g.) = 1 kilogram (Kg.)
1000 kilograms = 1 metric ton (T.)

Metric Equivalents

1 dm.^{3} = 1 l.
The weight of 1 cm.^{3} of water = 1 g.
The weight of 1 l. of water = 1 Kg.
The weight of 1 m.^{3} of water = 1 T.

Approximate Common Equivalents

1 m. = 39.37 in.
1 cm. = $\frac{2}{5}$ in.
1 Km. = $\frac{5}{8}$ mi.
1 Ha. = $2\frac{1}{2}$ acres
1 l. = 1.1 liquid quarts
1 Kg. = 2.2 lb.
1 T. = 2200 lb.

More Exact Equivalents

1 Km. = .62138 mi.
1 m. = 39.37 in.
1 l. = 1.0567 liquid qt. = .908 dry qt.
1 g. = 15.432 grains
1 Kg. = 2.205 lb.
1 T. = 2204.62 lb.
1 m.^{3} = 35.314 cu. ft.
1 hectare = 2.472 acres

SPECIFIC GRAVITIES

Silver	10.47
Gold	19.28
Platinum	21.50
Mercury	13.57
Steel	7.83
Cast iron	7.2
Bituminous coal	1.4
Ebony	1.33
Walnut	.67
Cork	.24
Sea water	1.03
Alcohol	.84
Ice	.92
Sulphur	2.00
Milk	1.032
Olive oil	.91
Brass	8.4

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